Merger rates in hierarchical models of galaxy formation

C. Lacey & S. Cole; MNRAS, 1993, 262, 627

C. Lacey & S. Cole; MNRAS, 1994, 271, 676

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Content

- Introduction & Basics
- Model & Formalism
- Some Results & Plots

Introduction

- large scale structure grows via gravitational instability from small-amplitude density fluctuations
- amplitudes of fluctuations decrease with increasing scale

$\Rightarrow$ small objects form first, merge and form larger objects

Density Fluctuations

$$\delta(x,t) \equiv \frac{\rho(x,t) - \rho_m}{\rho_m}$$

- ρ_m mean density
- $\delta(x,t) = \delta(x,t_0)D(t)/D(t_0)$
- D(t) linear growth factor
  $\Rightarrow$ depends on cosmology!

Critical Threshold

- extrapolate $\delta(x,t)$ according to linear theory to some time t_0

$$\delta(x,t_0) \equiv \delta_0(x) \equiv \delta(x)$$

- lower critical overdensity threshold $\delta_C(t)$ with increasing time
- $\delta_C(t)$ depends on cosmology
- objects collapse when

$$\delta(x) = \delta_C(t)$$

Smoothing

- smooth field over scale R

$$\delta(R,x) \equiv \int W_R\left(|x-y|\right)\delta\left(y\right)d^3y$$

- $W_R(r)$ is a window function / filter
- $\delta(R,x)$ gives weighted average of $\delta(x)$ over spheres of radius R

Fourier Space

$$f(x) \equiv \frac{1}{(2\pi)^3} \int \hat{f}(k) \exp(ik \cdot x) d^3k$$

$$\hat{f}(k) \equiv \int f(x) \exp(-ik \cdot x) d^3x$$

$$\Rightarrow \delta(R, x) = \frac{1}{(2\pi)^3} \int \hat{\delta}(k) \widehat{W}_R(k) \exp(ik \cdot x) d^3k$$

Variance of density field

$$\sigma^2(R) \equiv \left\langle \delta^2(R,x) \right\rangle_R - \left\langle \delta(R,x) \right\rangle_R^2$$

$$= \left\langle \delta^2(R,x) \right\rangle_R$$

$$= \int \widehat{W}_R^2(k) P(k) d^3k$$

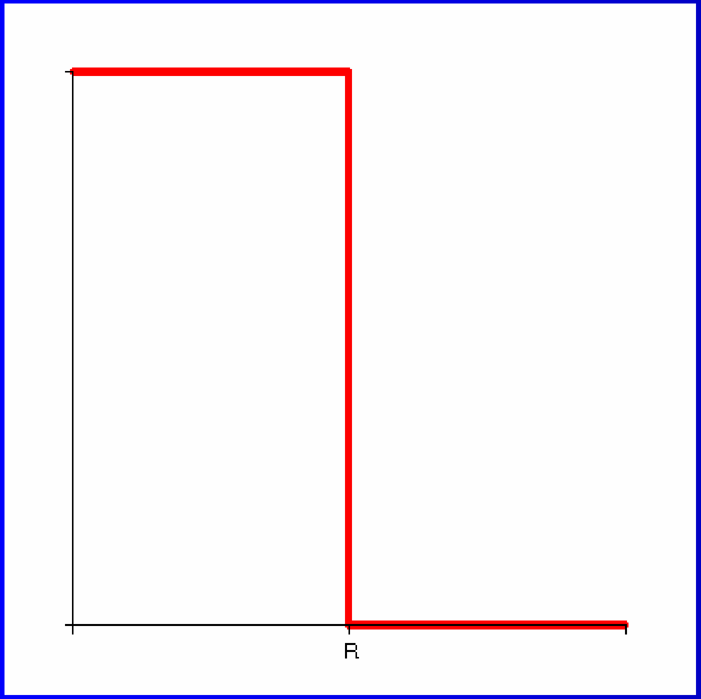
Power Spectrum

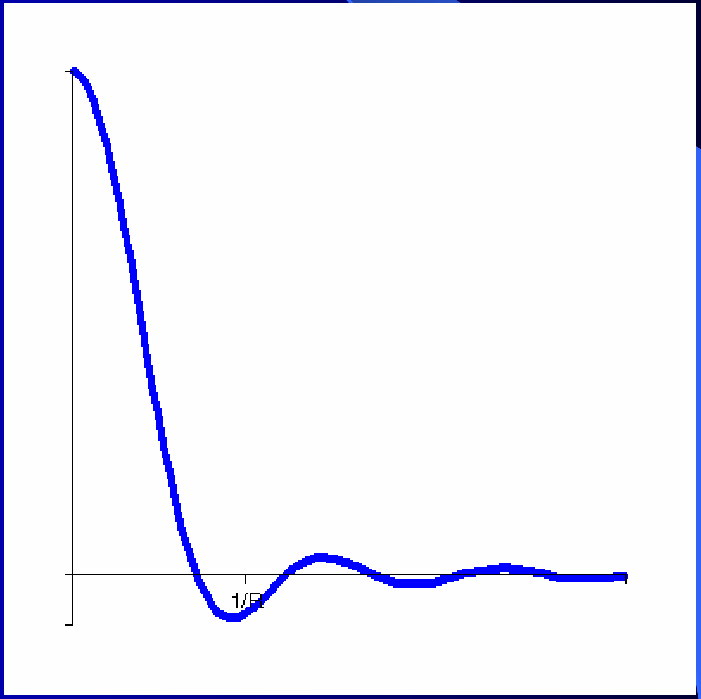
$$P(k) \equiv \frac{1}{(2\pi)^3} \left| \delta(k) \right|^2 \sim k^n$$

Top Hat Filter

$$W_R(r) = \begin{cases} 3/(4\pi R^3) & r < R \\ 0 & r > R \end{cases}$$

$$\widehat{W}_R(k) = \frac{3(\sin(kR) - (kR)\cos(kR))}{(kR)^3}$$

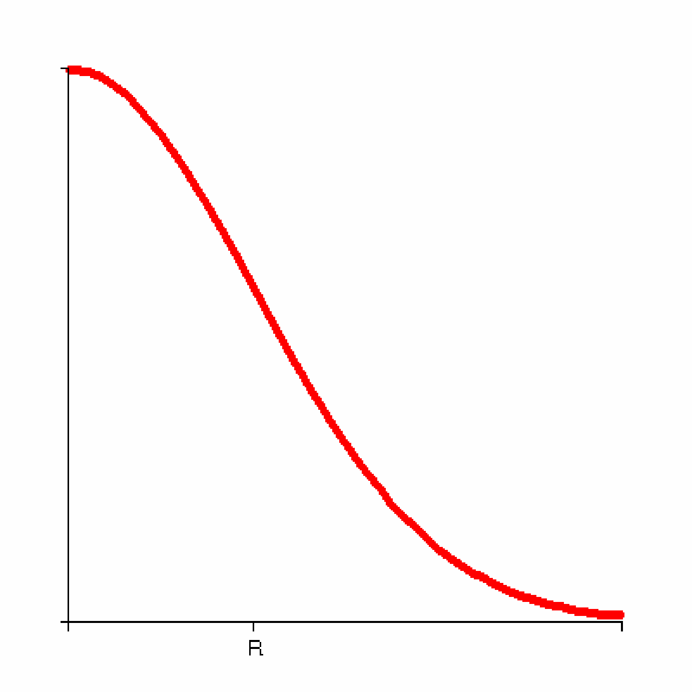


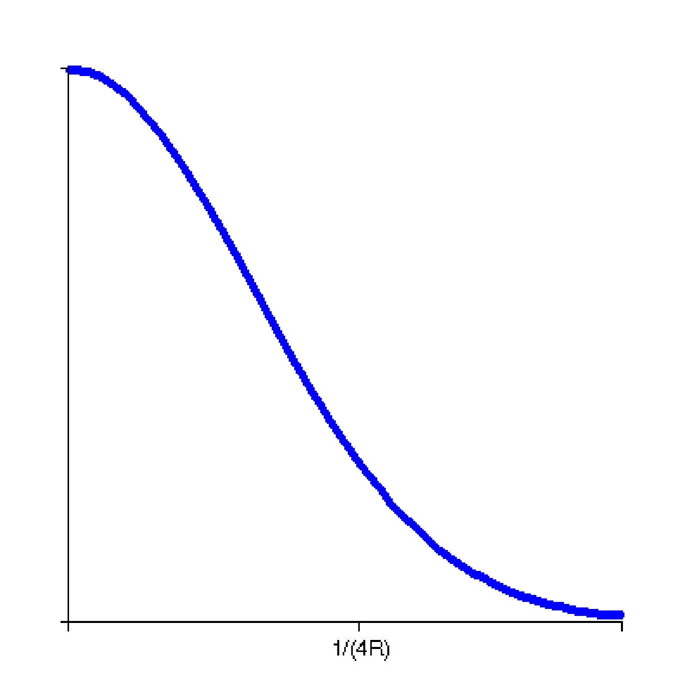


Gaussian Filter

$$W_R(r) = \frac{1}{(2\pi)^{3/2} R^3} \exp\left(-\frac{r^2}{2R^2} \right)$$

$$\widehat{W}_R(k) = \exp\left(-\frac{k^2 R^2}{2} \right)$$

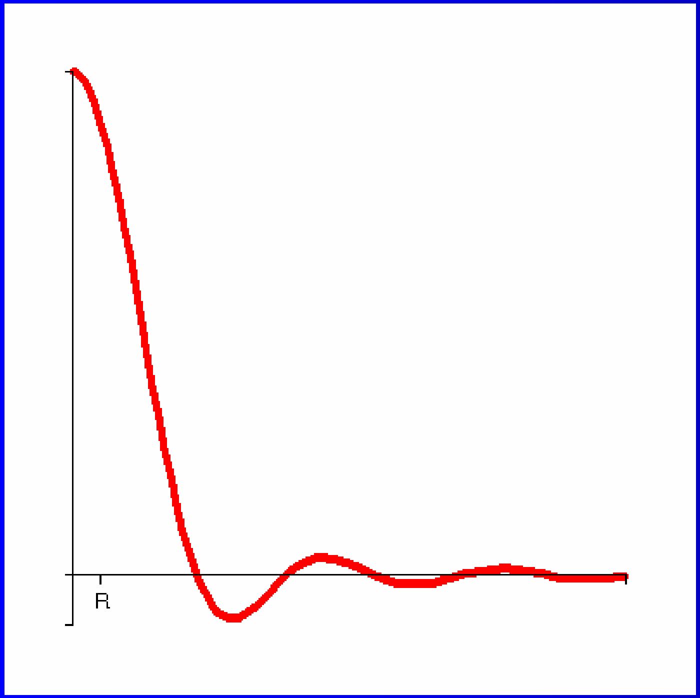


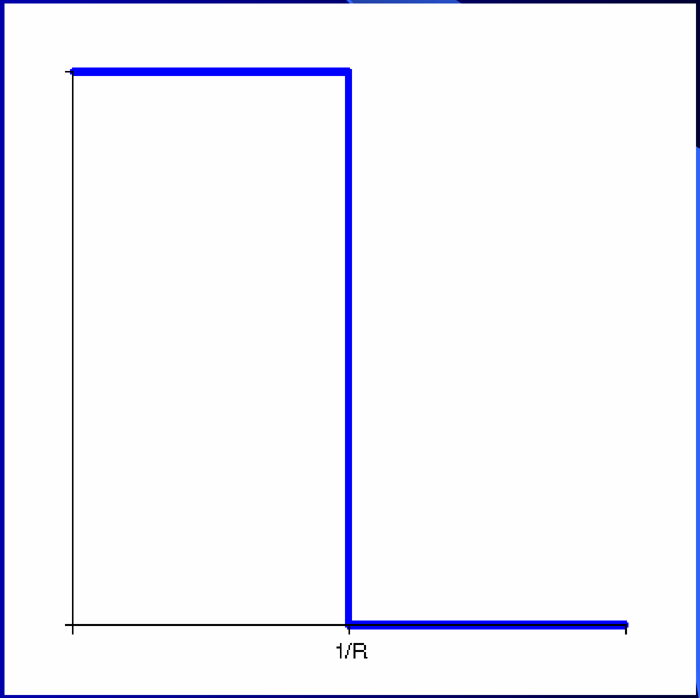


Sharp k-Space Filter

$$W_R(r) = \frac{\sin(r/R) - (r/R)\cos(r/R)}{2\pi^2 r^3}$$

$$\widehat{W}_R(k) = \begin{cases} 1 & k < 1/R \\ 0 & k > 1/R \end{cases}$$





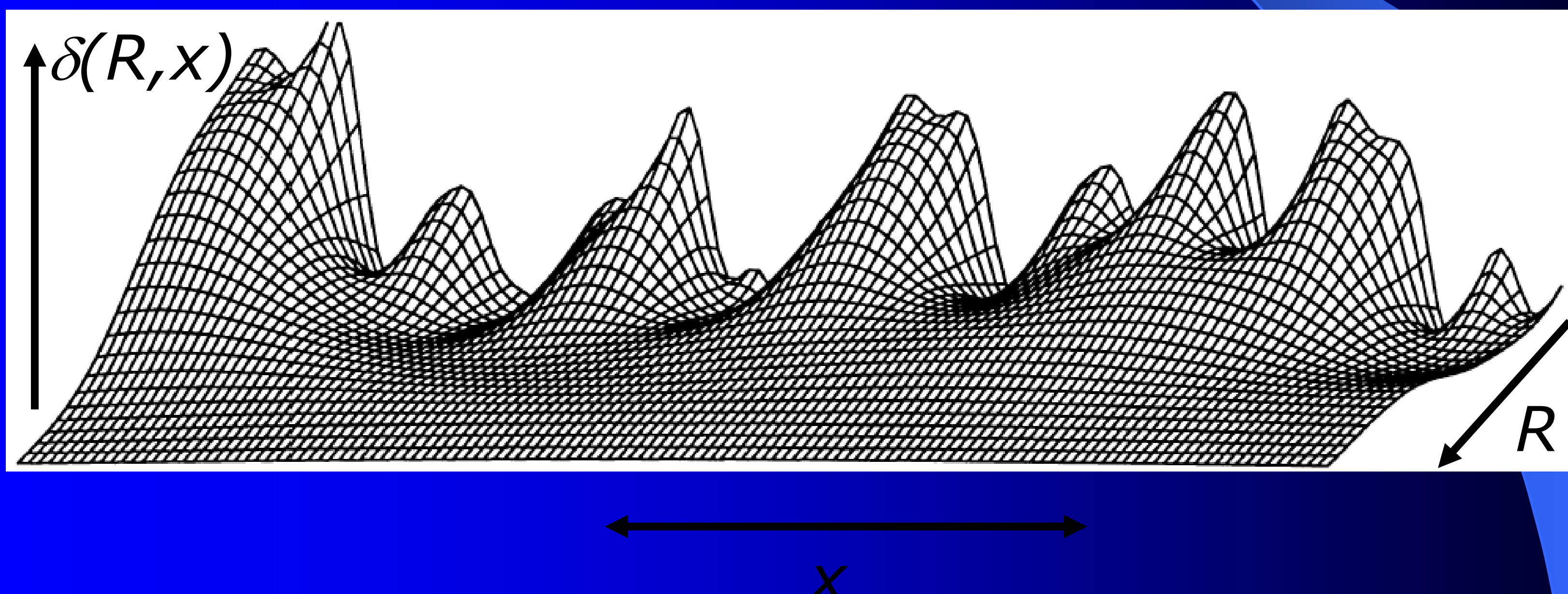
R-M Relation

- each filter has a natural volume
- Top Hat Filter: $V(R) = (4\pi/3)R^3$
- Gaussian Filter: $V(R) = (2\pi)^{3/2}R^3$
- Sharp k-Space Filter: $V(R) = 6\pi^2 R^3$
- R-M relation:

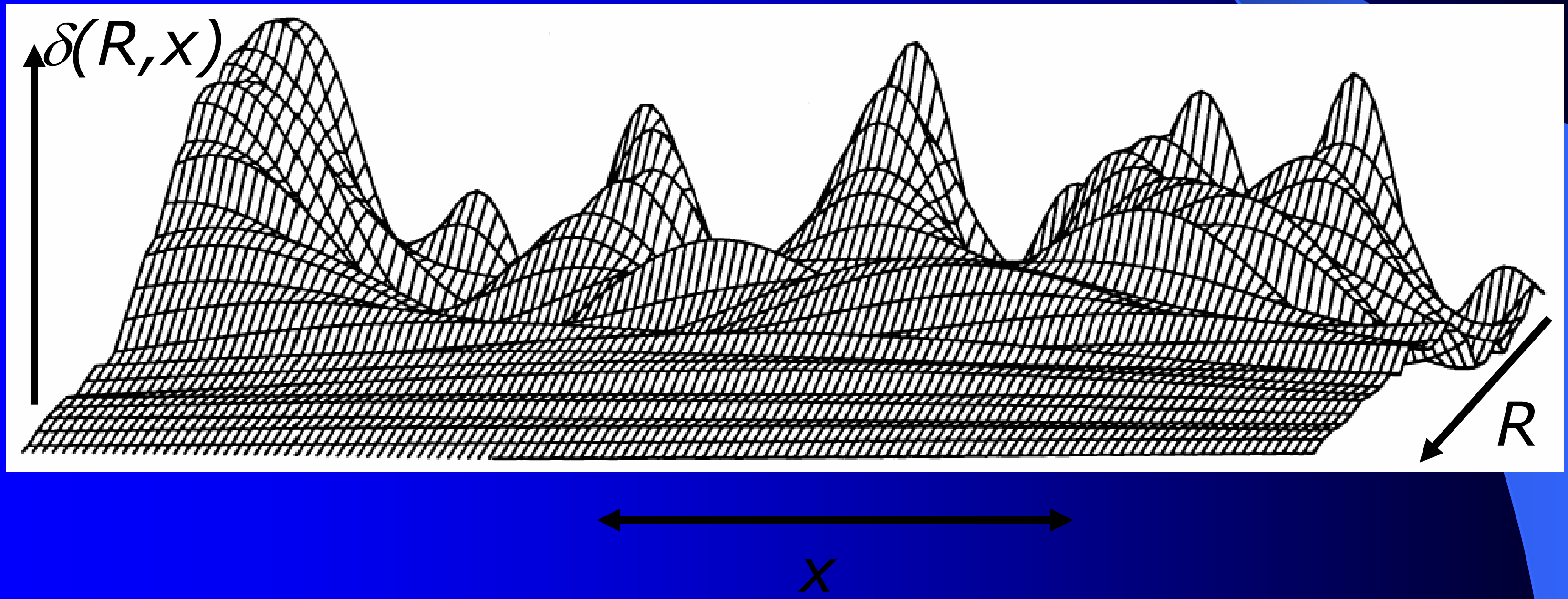
$$M(R) \equiv \rho_m V(R)$$

$$\Rightarrow \delta(R, x) = \delta(M, x)$$

Gaussian Filter in 1D



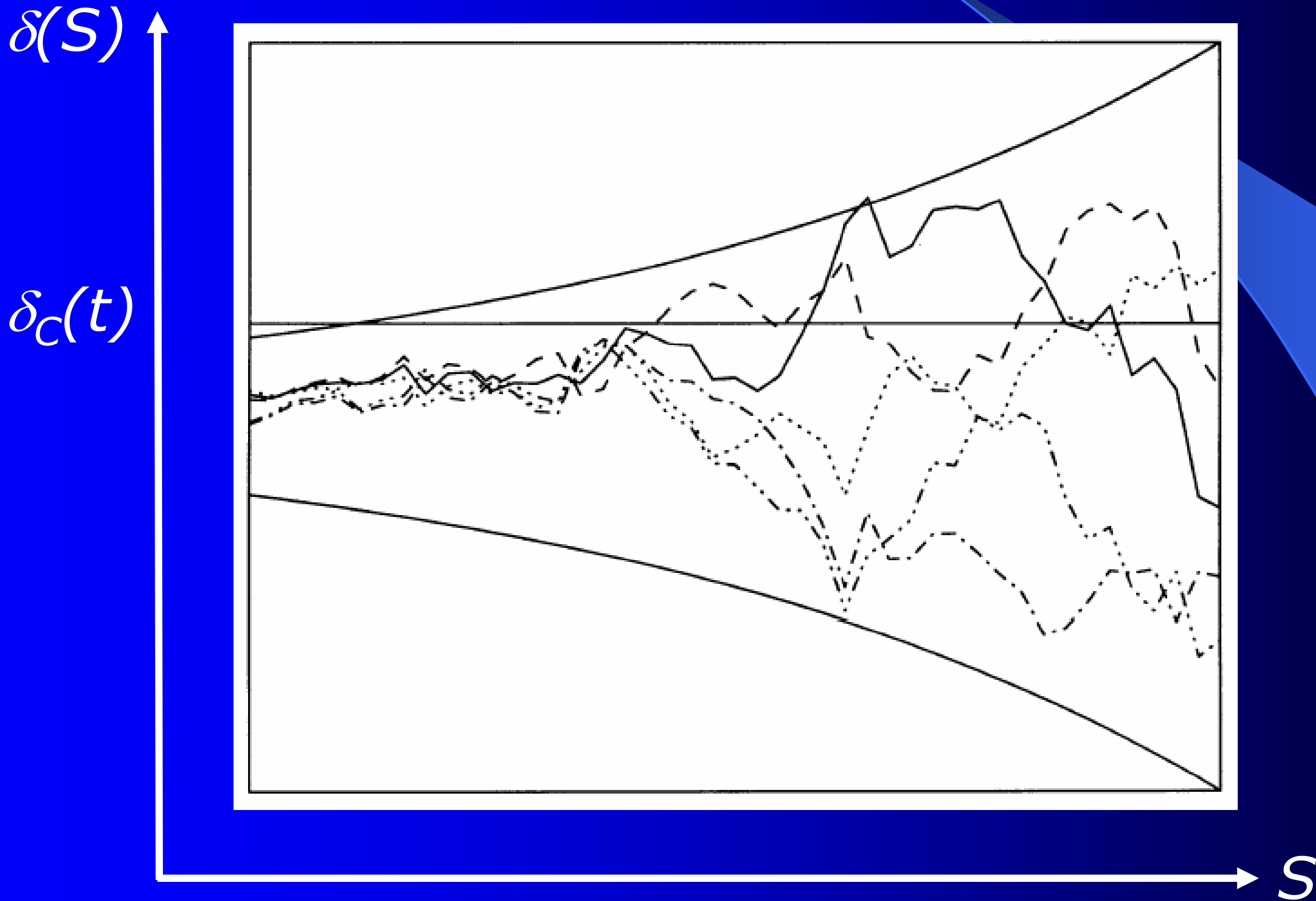
Sharp k-Space Filter in 1D



Random Walk I

- if sharp k-space filter is used
  $\Rightarrow$ trajectories $\delta(R,x)$ make random walk for varying R
  $\Rightarrow$ analytical solution possible
- always a new shell in k-space contributes $\Rightarrow$ uncorrelated
- write $\delta(S)$ for a fixed location x with $S \equiv \sigma^2(R) = \sigma^2(M)$
- for $P(k) \sim k^n \Rightarrow S \sim M^{-(n+3)/3}$

Random Walk II



Random Walk III

- distribution of trajectories is gaussian without barrier absorption

$$P(\delta, S) = \frac{1}{\sqrt{2\pi S}} \exp\left(-\frac{\delta^2}{2S} \right)$$

- $Q(\delta, S)$ distribution / number density of trajectories with barrier absorption

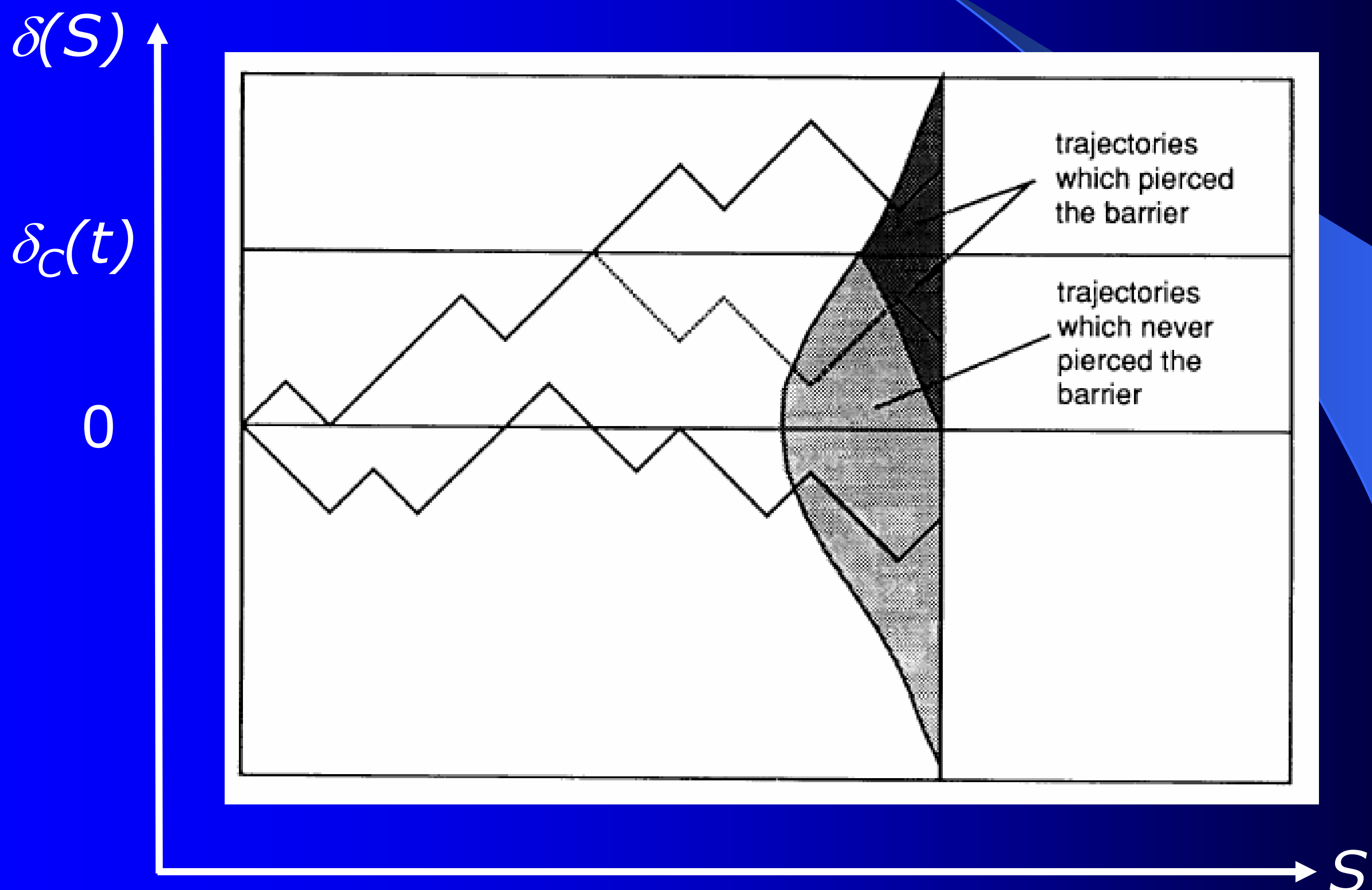
Random Walk IV

- Q obeys a diffusion equation

$$\frac{\partial Q}{\partial S} = \frac{1}{2}\frac{\partial^2 Q}{\partial \delta^2}$$

- All trajectories begin at $S=0$ and $\delta=0$ and diffuse away

Trajectories above $\delta_c(t)$



Number density

- Unique solution to the diffusion equation with absorbing boundary condition

$$Q(\delta, S, \omega) = \frac{1}{\sqrt{2\pi S}} \left\{ \exp\left(-\frac{\delta^2}{2S} \right) - \exp\left(-\frac{(\delta - 2\omega)^2}{2S} \right) \right\}$$

- $\omega \equiv \delta_C(t)$
- $\delta < \delta_C(t)$

Absorption Probability

$$f(S,\omega) = -\frac{\partial}{\partial S}\int_{-\infty}^{\omega} Q\,d\delta = -\left[\frac{1}{2}\frac{\partial Q}{\partial \delta}\right]_{-\infty}^{\omega}$$

$$= \frac{\omega}{\sqrt{2\pi S^3}}\exp\left(-\frac{\omega^2}{2S}\right)$$

$$f(M,t) = \frac{\delta_c(t)}{\sqrt{2\pi\sigma^3(M)}}\exp\left(-\frac{\delta_c^2(t)}{2\sigma^2(M)}\right)$$

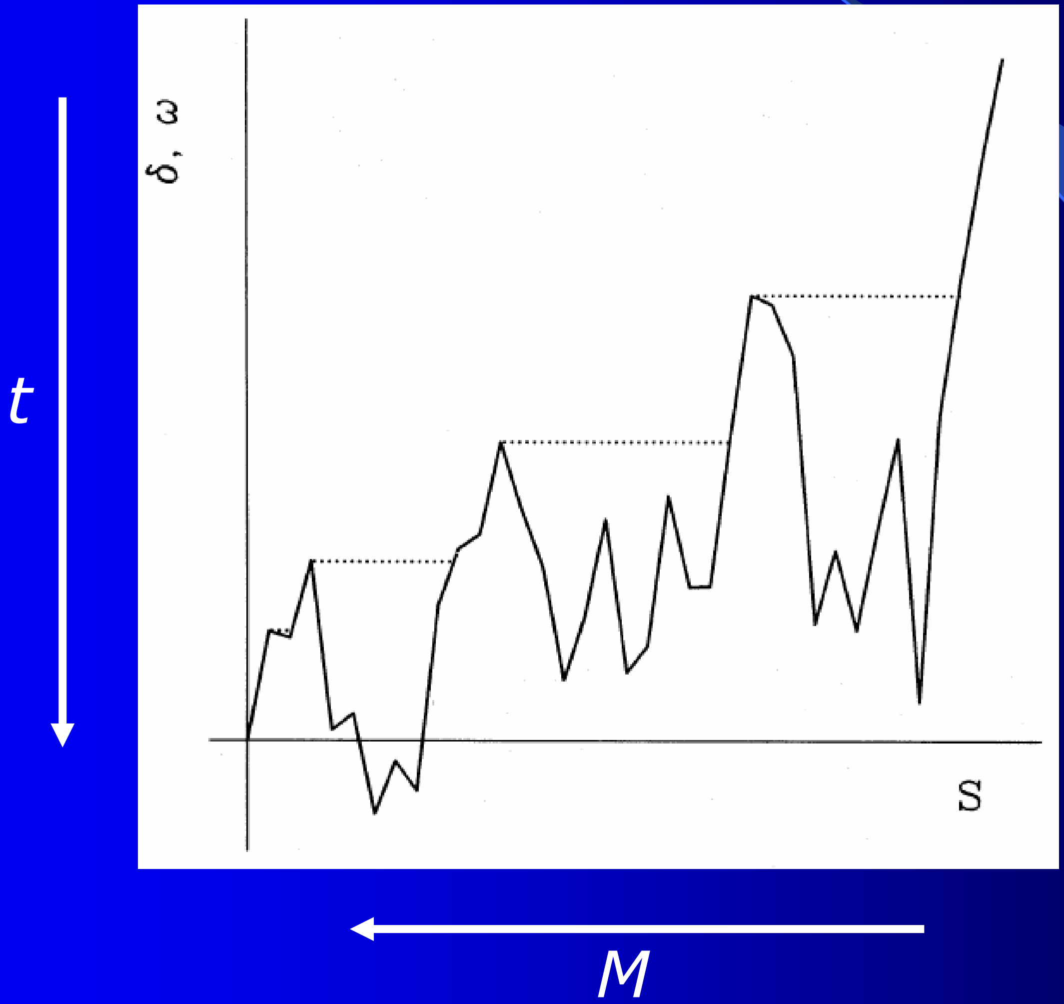
$\Rightarrow$ Fraction of Mass associated with halos of Mass *M*

Number density of halos of Mass M

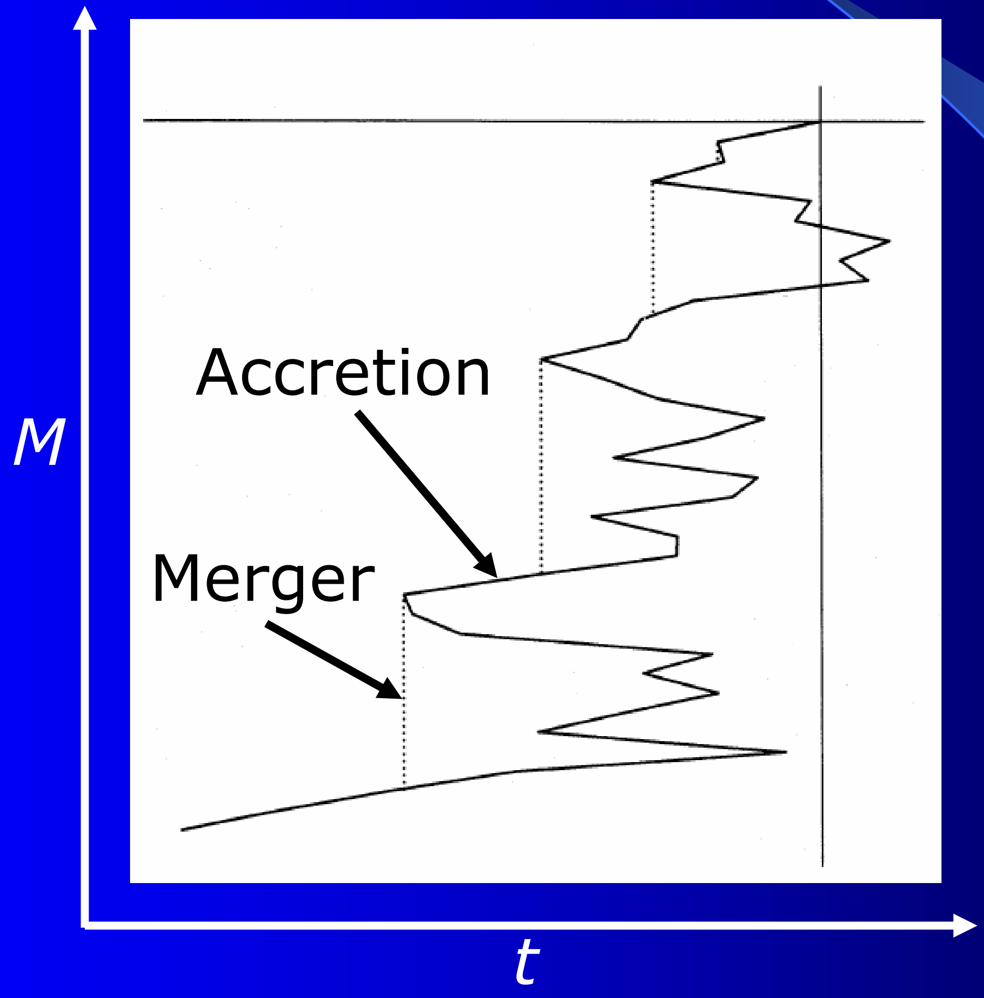
$$n(M,t) = \frac{\rho_m}{M} f_S(S,\omega) \left| \frac{dS}{dM} \right|$$

$$= \sqrt{\frac{2}{\pi}} \frac{\rho_m}{M^2} \frac{\delta_c(t)}{\sigma(M)} \left| \frac{d\ln(\sigma)}{d\ln(M)} \right| \exp\left(-\frac{\delta_c^2(t)}{2\sigma^2(M)} \right)$$

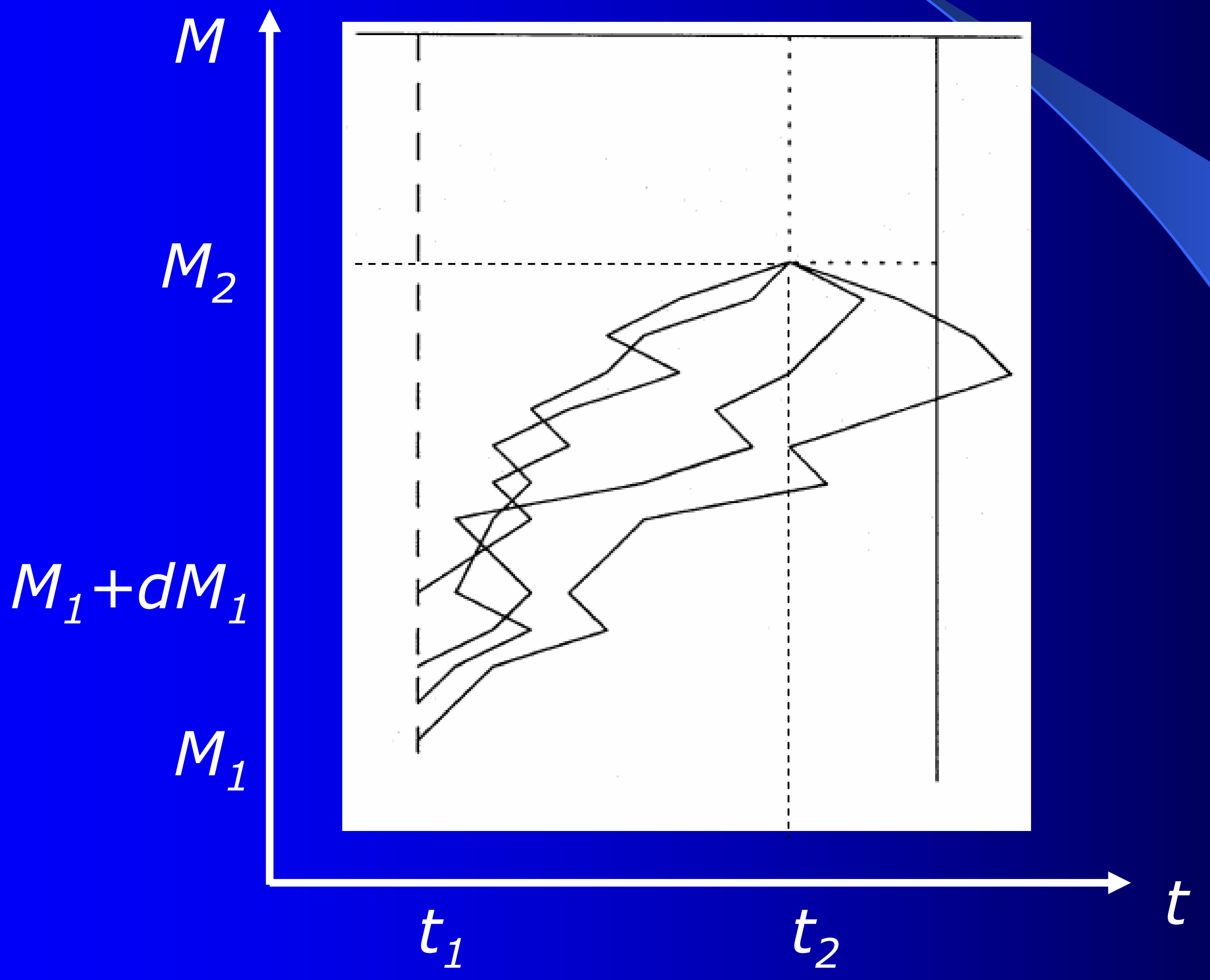
Merger History I



Merger History II



Merger Rate I



Merger Rate II

$$f(M_1,t_1|M_2,t_2)=\frac{\left(\delta_{C1}-\delta_{C2}\right)}{\sqrt{2\pi}\left(\sigma_1^2-\sigma_2^2\right)^{3/2}}\left|\frac{d\sigma_1^2}{dM_1}\right|$$

$$\times\exp\left(-\frac{\left(\delta_{C1}-\delta_{C2}\right)^2}{2\left(\sigma_1^2-\sigma_2^2\right)}\right)$$

$$f(M_2,t_2|M_1,t_1)=\frac{\delta_{C2}\left(\delta_{C1}-\delta_{C2}\right)}{\sqrt{2\pi}\delta_{C1}}\left(\frac{\sigma_1^2}{\sigma_2^2\left(\sigma_1^2-\sigma_2^2\right)}\right)^{3/2}\left|\frac{d\sigma_2^2}{dM_2}\right|$$

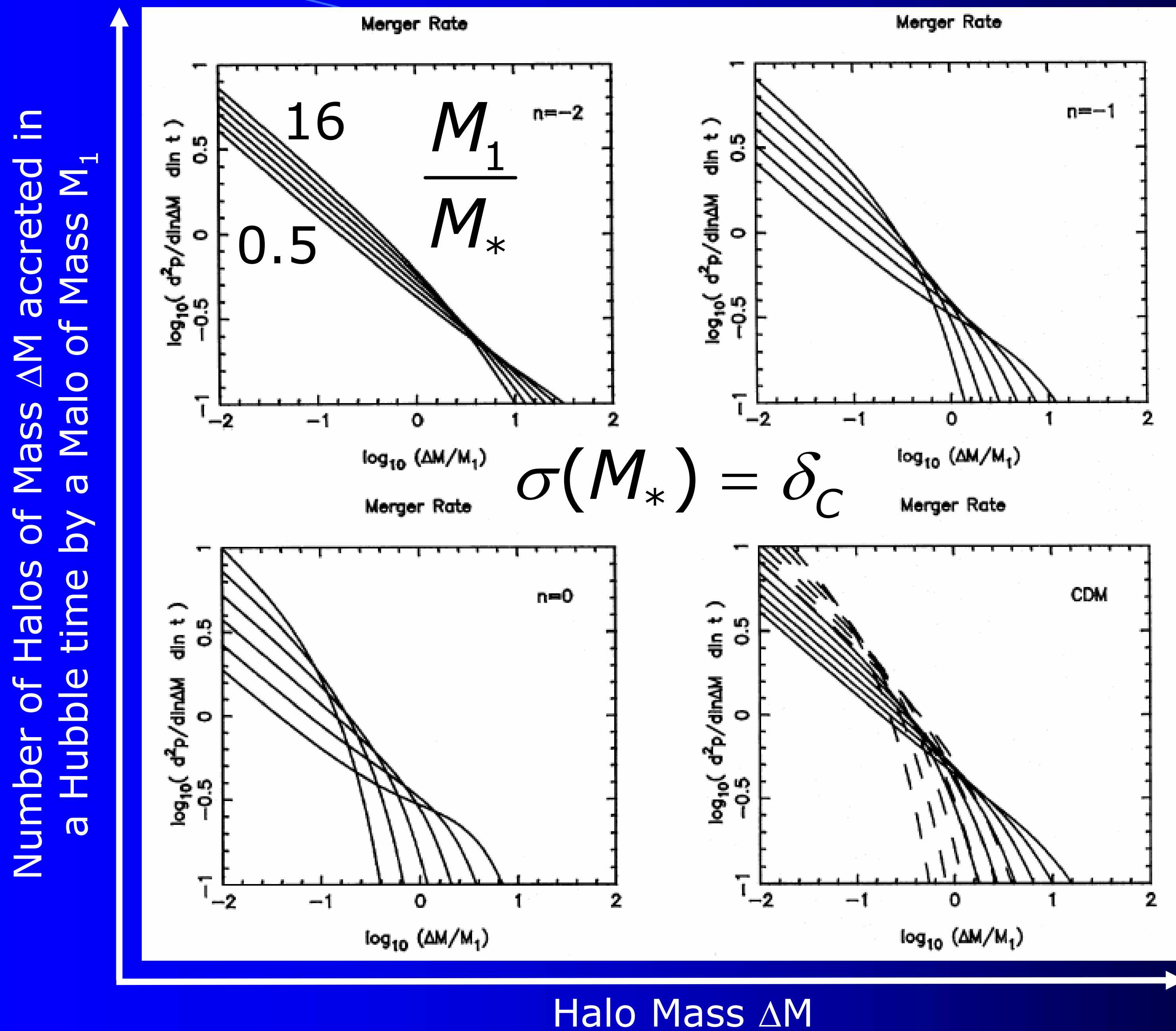
$$\times\exp\left(-\frac{\left(\delta_{C1}\sigma_1^2-\delta_{C2}\sigma_2^2\right)^2}{2\sigma_1^2\sigma_2^2\left(\sigma_1^2-\sigma_2^2\right)}\right)$$

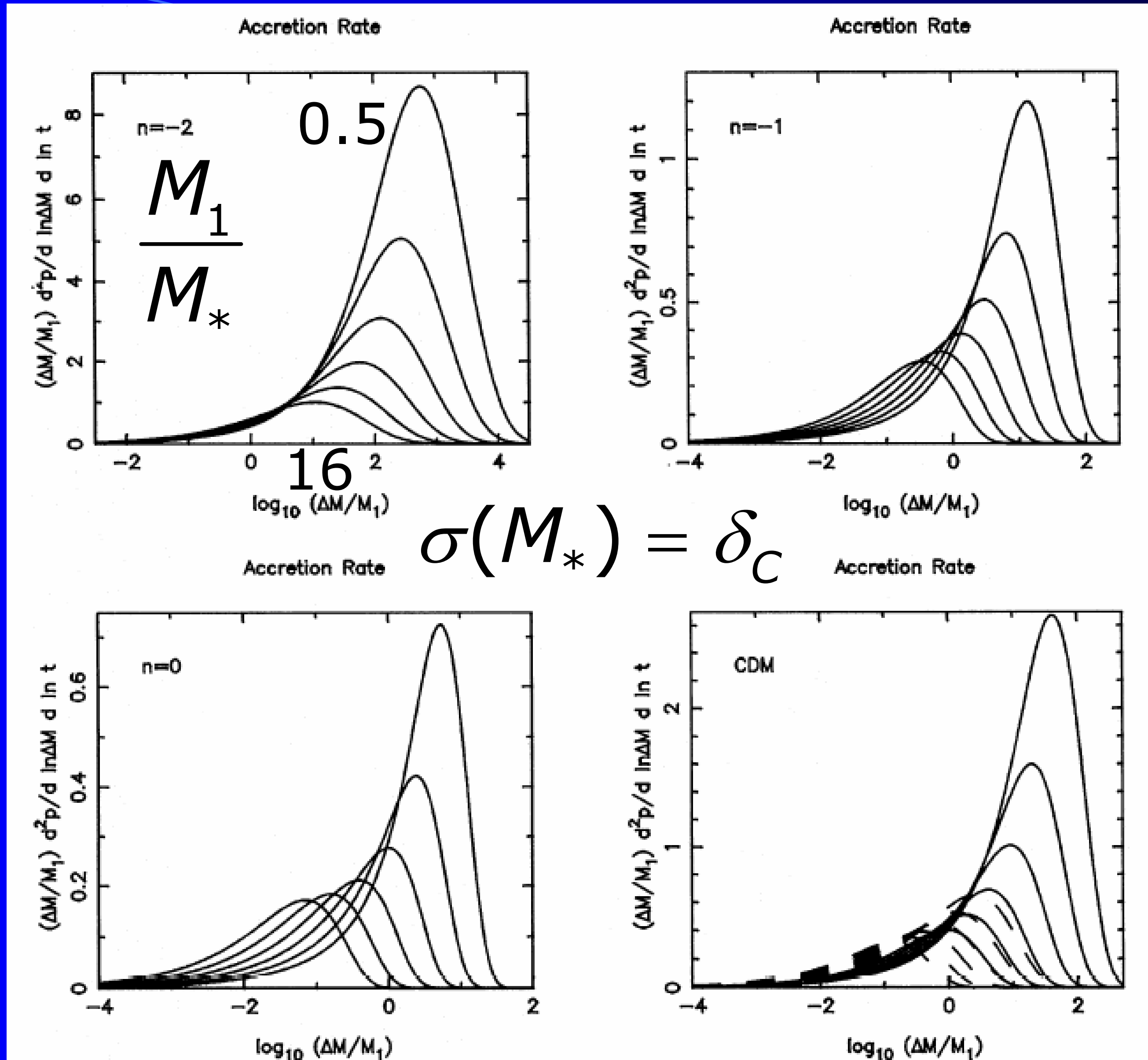
Merger Rate III

- take the limit of $f(M_2,t_2|M_1,t_2)$

$$t_2 \rightarrow t_1 \quad \Delta M \equiv M_2 - M_1$$

- you get the instantaneous merger rate $p(M_1 \rightarrow M_2|t)$!


Merger Rate
n=−2
16
0.5
$\frac{M_1}{M_*}$
$\log_{10}(\, d^2p/d\ln\Delta M \;\; d\ln t\,)$
$\log_{10}\,(\Delta M/M_1)$
Merger Rate
n=−1
$\sigma(M_*) = \delta_c$
Merger Rate
n=0
Merger Rate
CDM
Number of Halos of Mass ΔM accreted in a Hubble time by a Malo of Mass M_1
Halo Mass ΔM



Fractional Mass accreted per Hubble time

Halo Mass ΔM