An optimum time-stepping Scheme for N-body Simulations

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Content

- General idea and description
- Implementation within tree-code
- Behaviour and tests

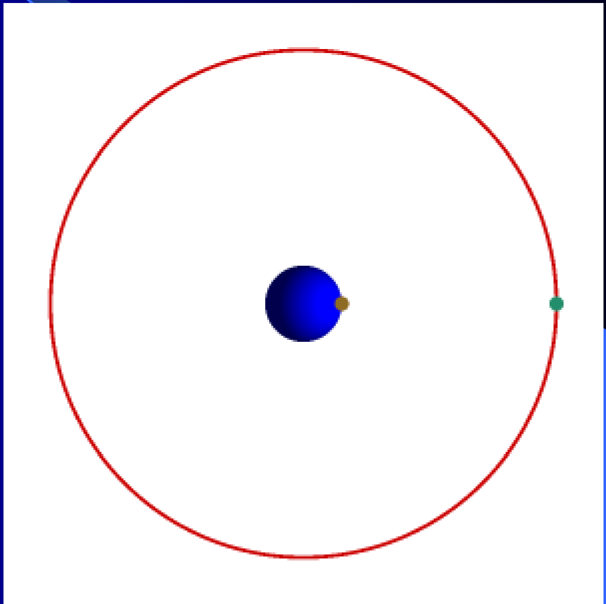
General Idea

- What is the natural time-scale?

Dynamical Time

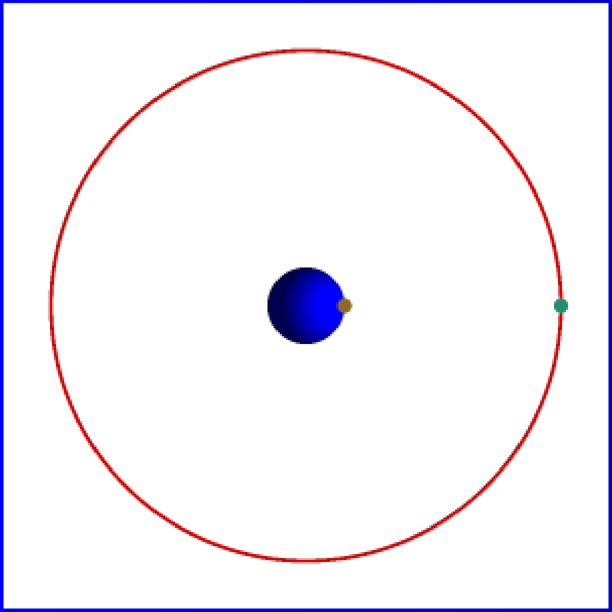
$$T_{dyn}(r) = \frac{2\pi}{\sqrt{G\rho_{enc}(r)}}$$

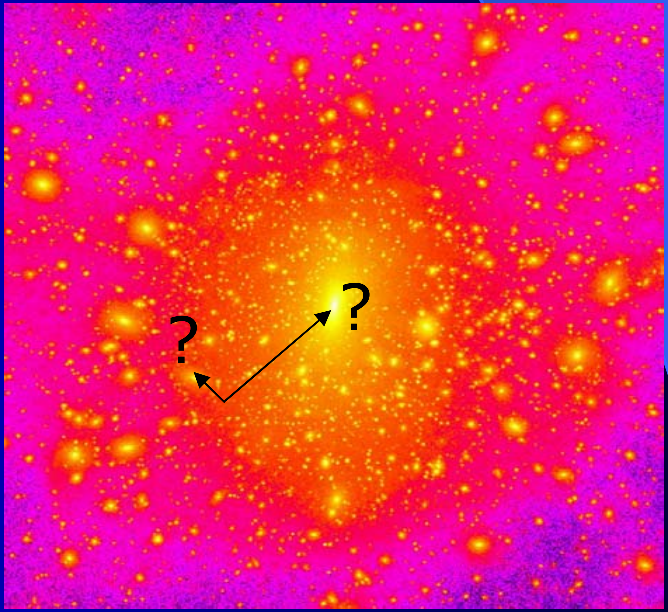
$$\rho_{enc}(r) \equiv M(r) / r^3$$



Dynamical Time-Stepping

- Natural choice $\Delta T \sim T_{dyn}$
- But what sets the dynamical time?





Two Regimes

- Mean field regime
  - global potential
  - no short range contributions

  $\Rightarrow \rho_{enc}$ set by global structures
- Gravitational scattering regime
  - large angel scattering
  - 2-body orbits with $e \rightarrow 1$

  $\Rightarrow \rho_{enc}$ set by local structures

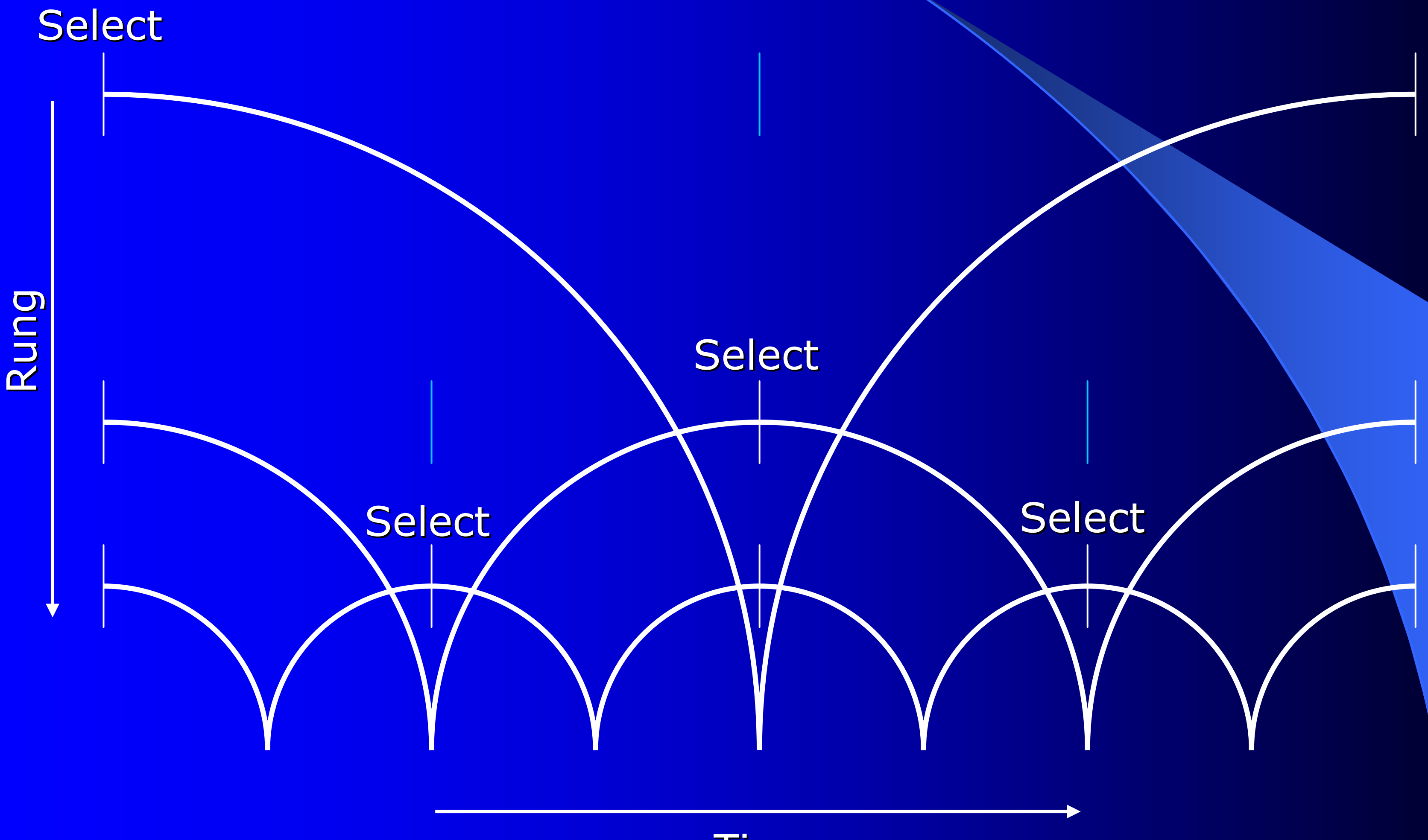
Implementation

- Block time-steps

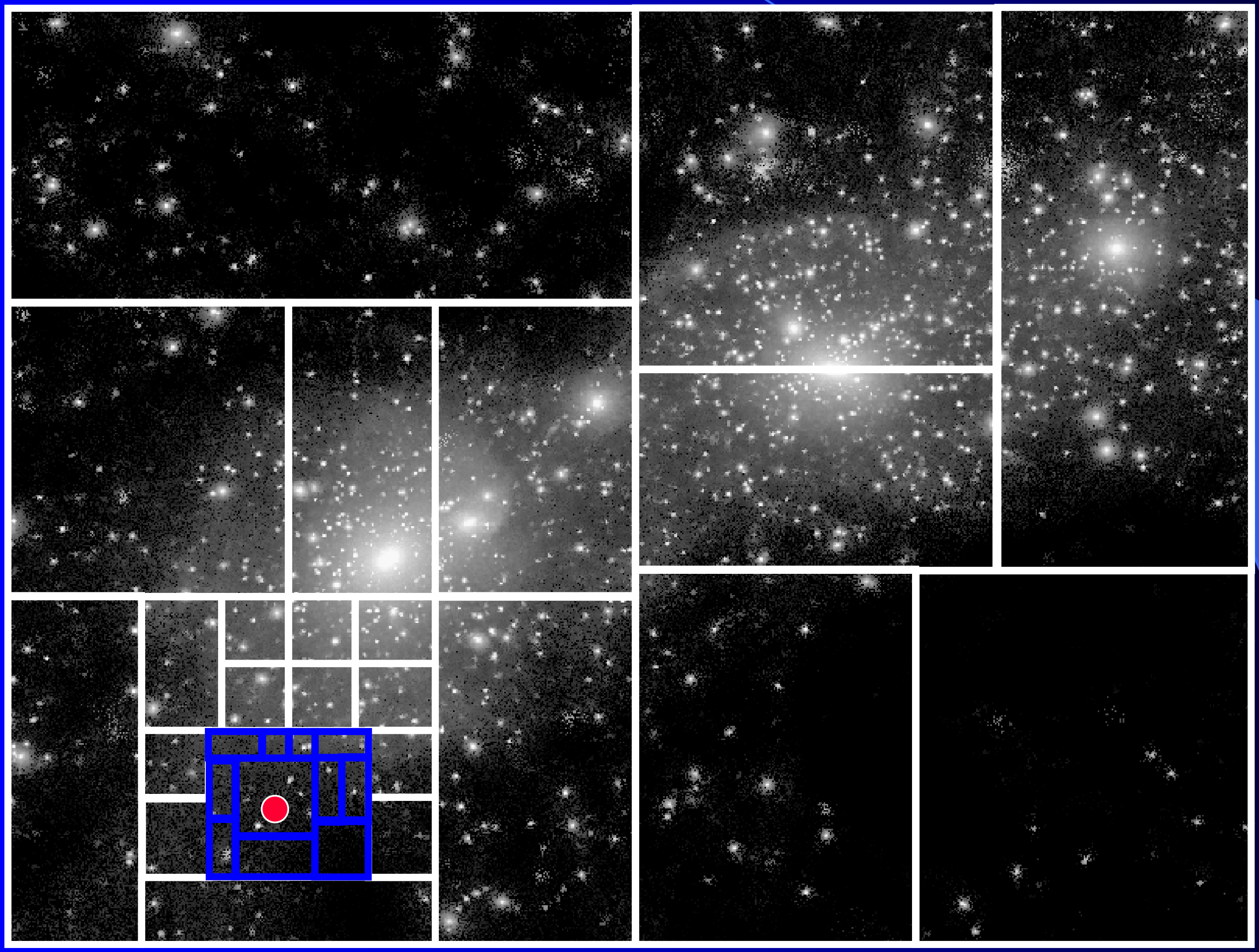
$$\Delta T = \frac{T_0}{2^n} \leq \eta_D \frac{1}{\sqrt{G\rho_{enc}(r)}}$$

- η_D is free parameter $\Rightarrow$ controls number of time-steps per orbit

Kick-Drift-Kick Leapfrog



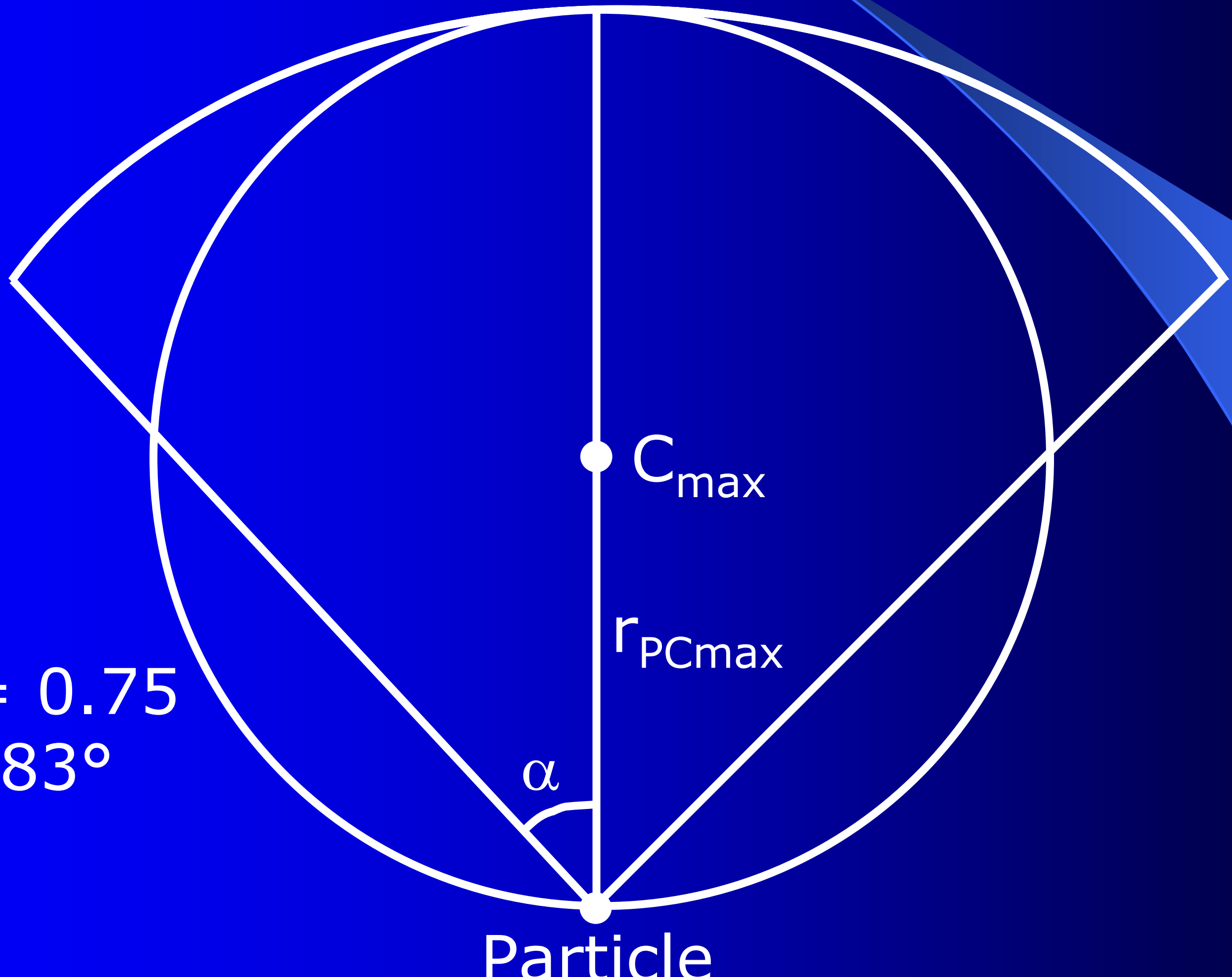
Mean Field



Mean Field Alogrithm I

- Calculate $\rho_{enc} = M_C / r_{PC}^3$ for each cell
- Pick out the top 0.5 % values
- Sum up ρ_{enc} values within viewing cone
- Set maximum as $\rho_{enc,MF}$
- Add local density ρ_{local}

Mean Field Algorithm II

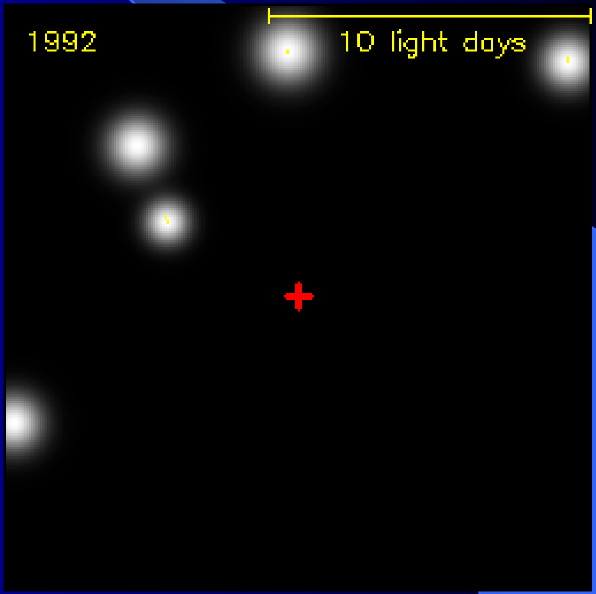


$\cos(\alpha) = 0.75$

$\Rightarrow 2\alpha \approx 83°$

Gravitational Scattering

- We also want to follow 2-body orbits with $e \to 1$ correctly (elliptic & hyperbolic)
- gravitational scattering
- new time-stepping is starting point but…



Eccentricity Correction I

- PKDGRAV: Kick-Drift-Kick Leapfrog
- Approximate Hamiltonian

$$H_A = H_0 + \Delta T^2 H_2 + \Delta T^4 H_4 + O(\Delta T^6)$$

$$H_0 = H_D + H_K = H$$

$$H_2 = 1/12 \{\{H_K, H_D\}, H_D\}$$
$$-1/24 \{\{H_D, H_K\}, H_K\}$$

Eccentricity Correction II

- Plug in Hamiltonian for 2-body problem and evaluate at periapsis of orbit with symmetrised time-step

$$\Rightarrow E_2^{peri} = \Delta T^2 H_2 = \frac{1}{24}\frac{(1+2e)}{(1-e)}\frac{\eta_D^2 G M_1 M_2}{a}$$

GS Algorithm

- Use symmetrised ρ_{enc}!

$$\rho_{enc,GS} = \frac{(1+2e)}{(1-e)} \frac{M_P + M_I}{r_{PP}^3}$$

- Similar to expression for orbital time in 2-body problem

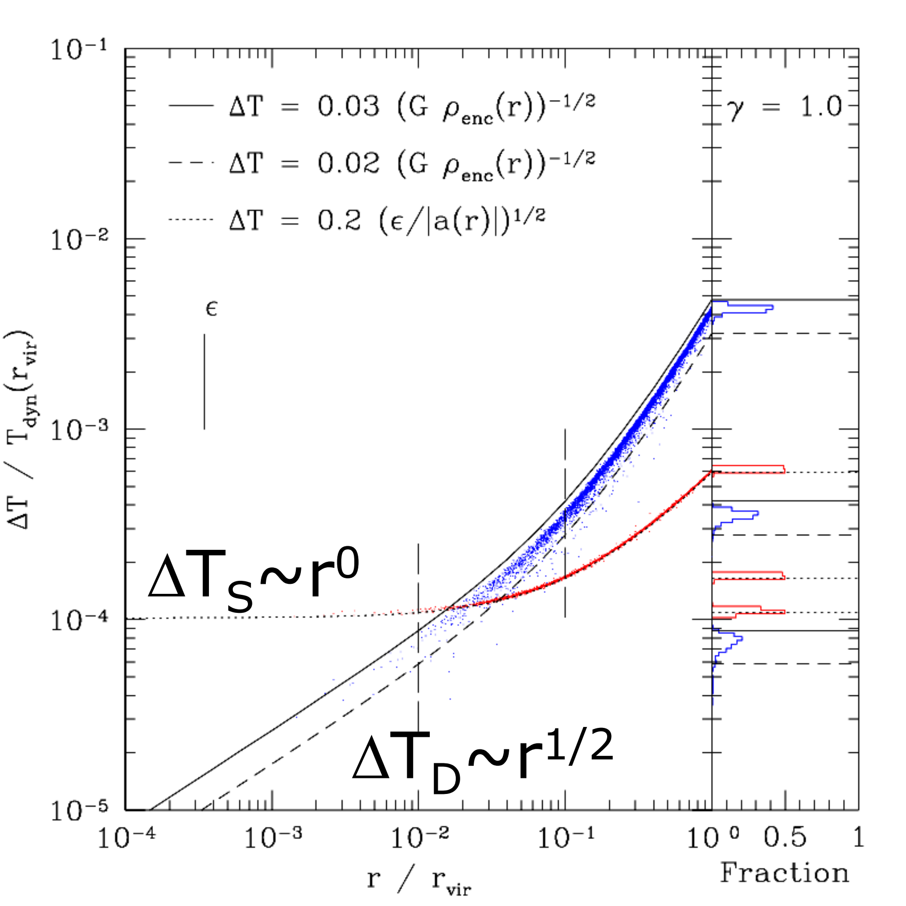
Standard Criterion

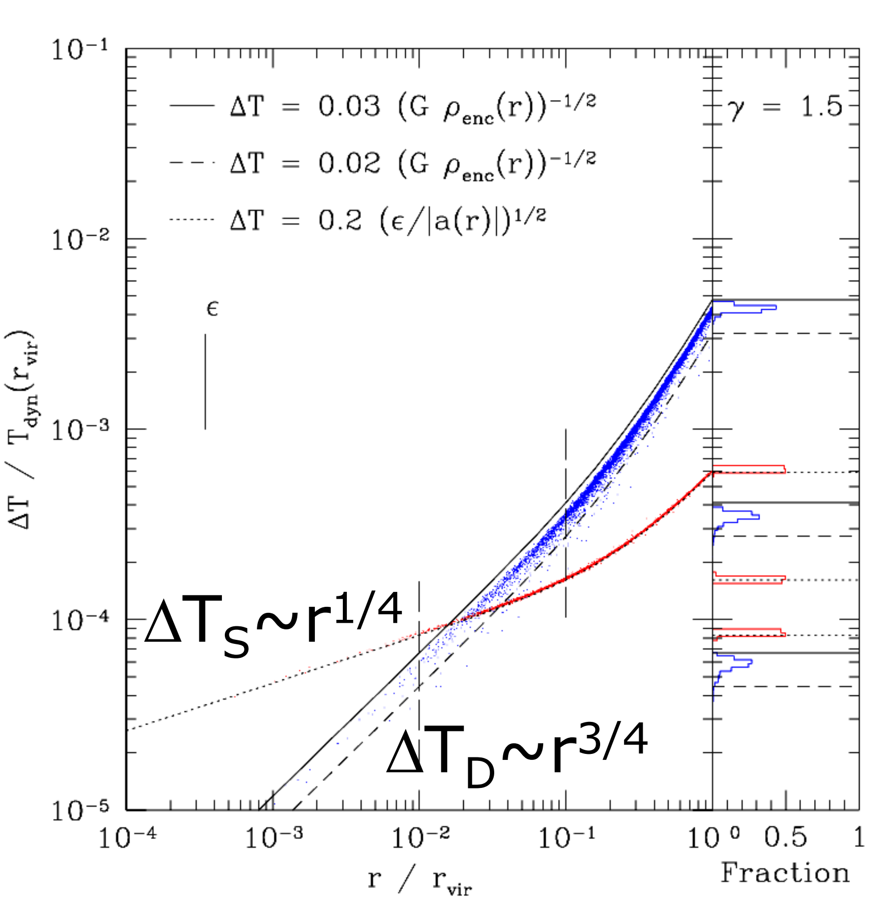
- Standard criterion used in PKDGRAV and GADGET

$$\Delta T = \frac{T_0}{2^n} \leq \eta_s \sqrt{\frac{\varepsilon}{a(r)}}$$

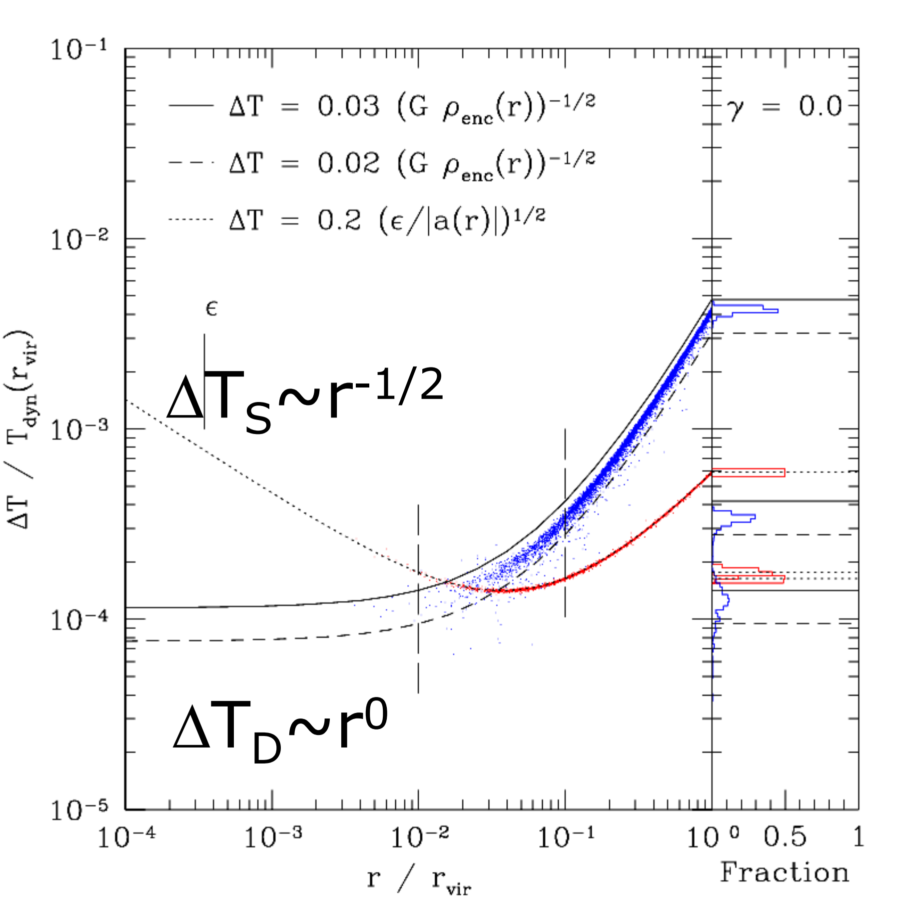
- Ad-hoc criterion
  $\Rightarrow$ no physical motivation!
- Dependence on artificial simulation parameter softening length ε

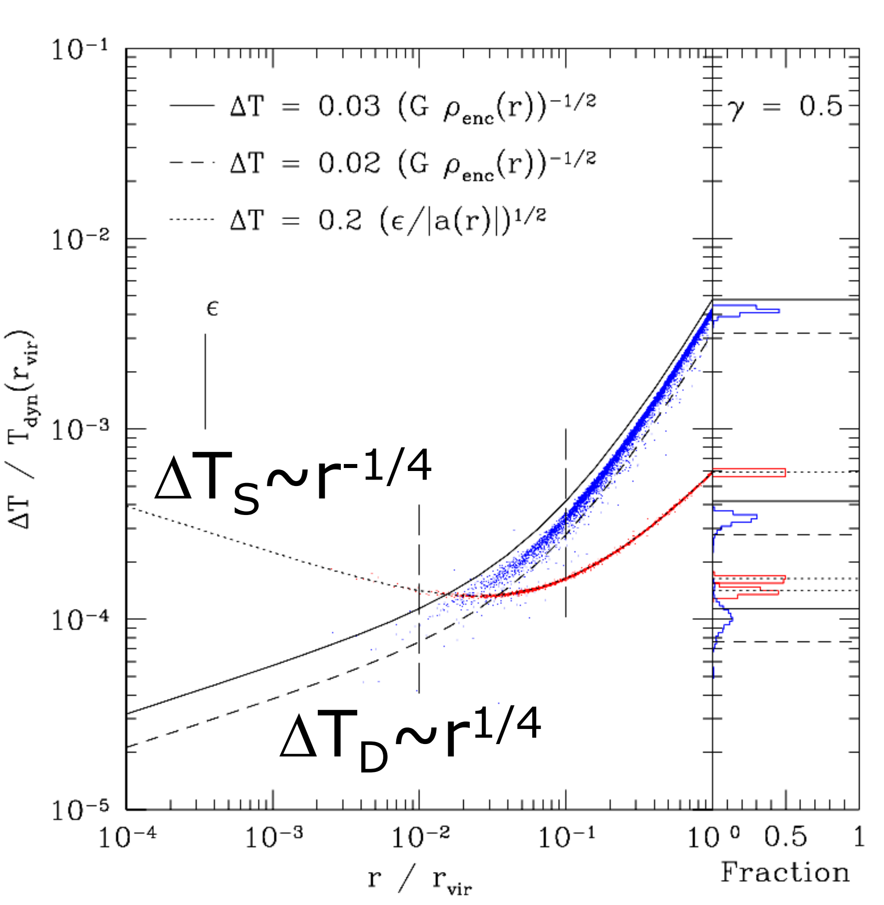
Comparison I



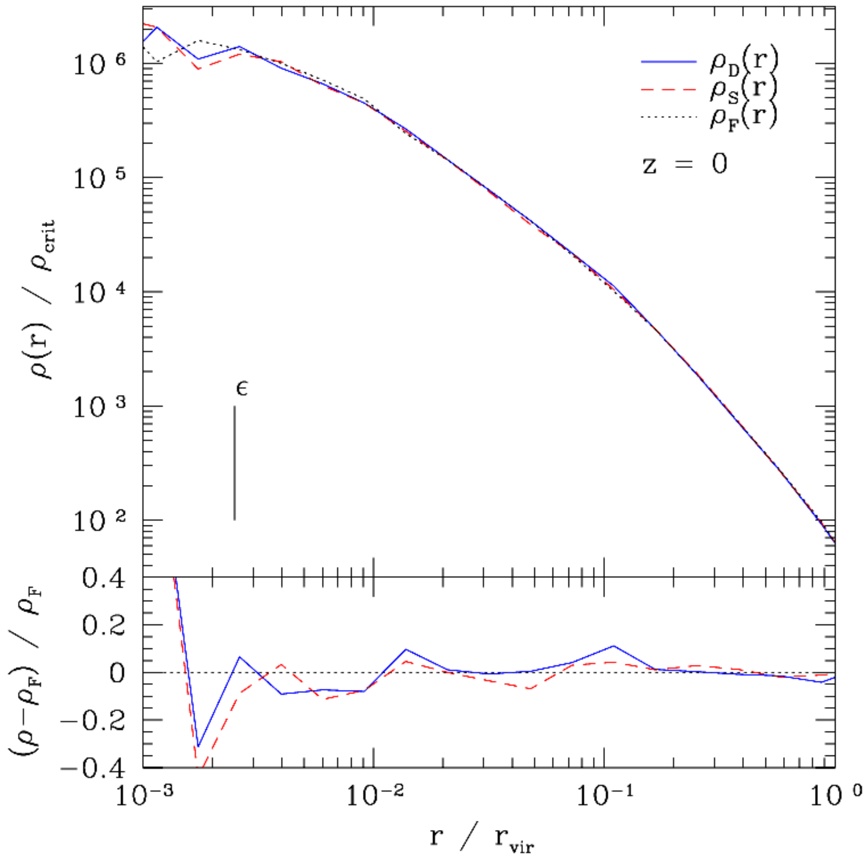


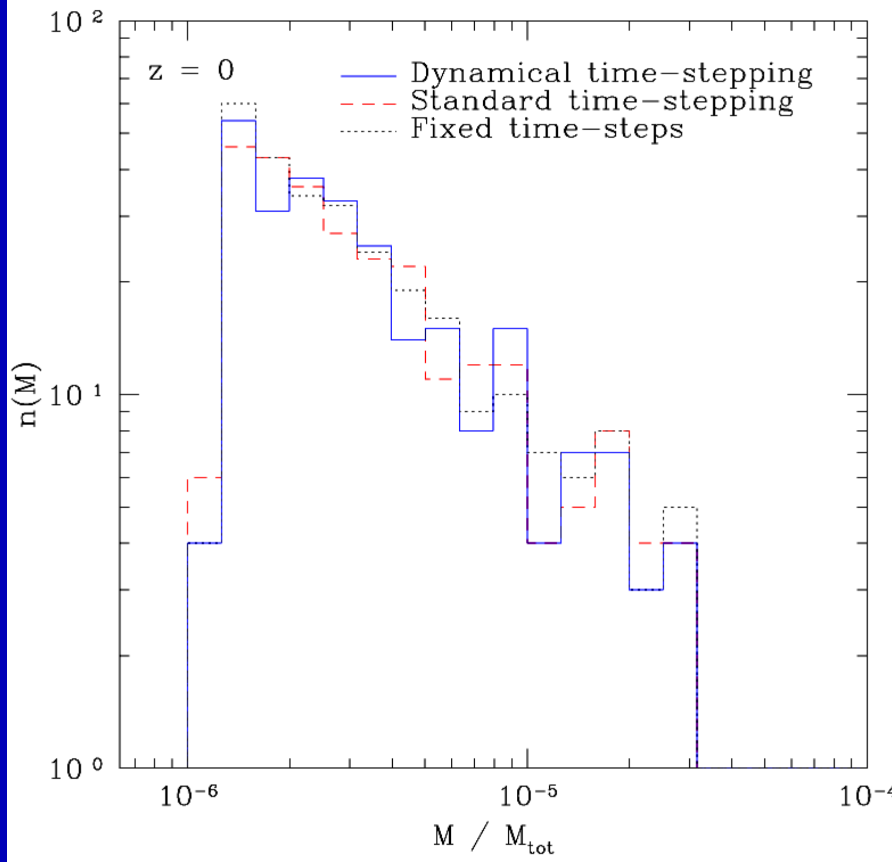
Comparison II



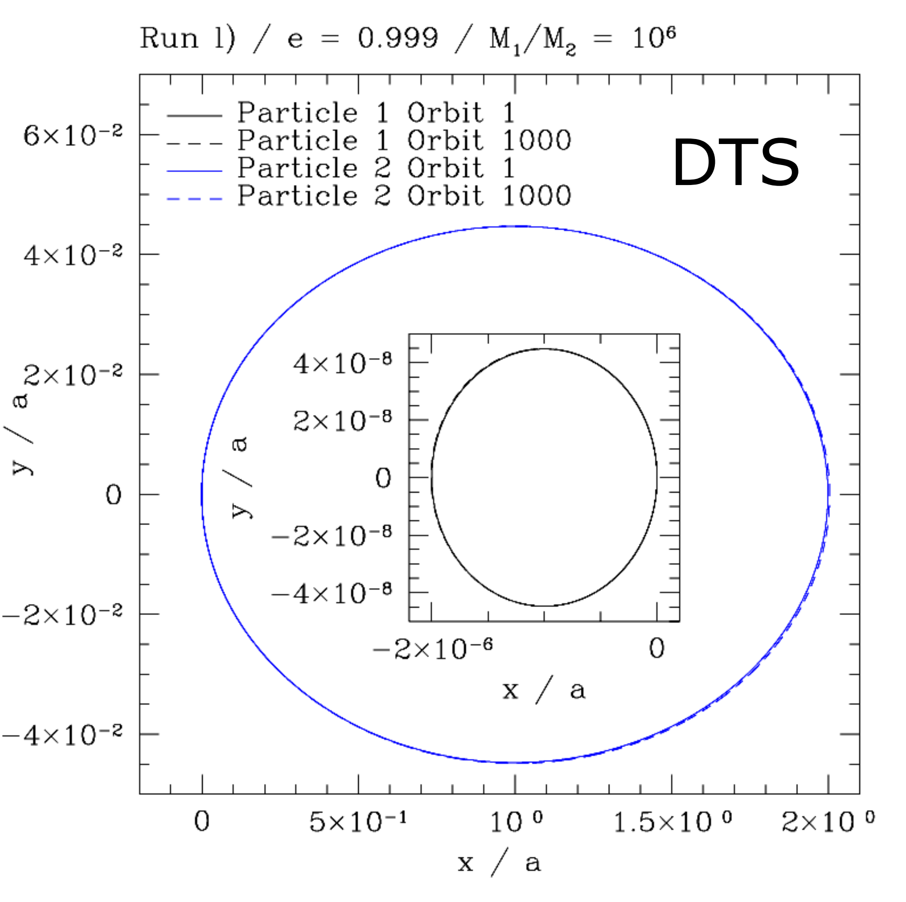


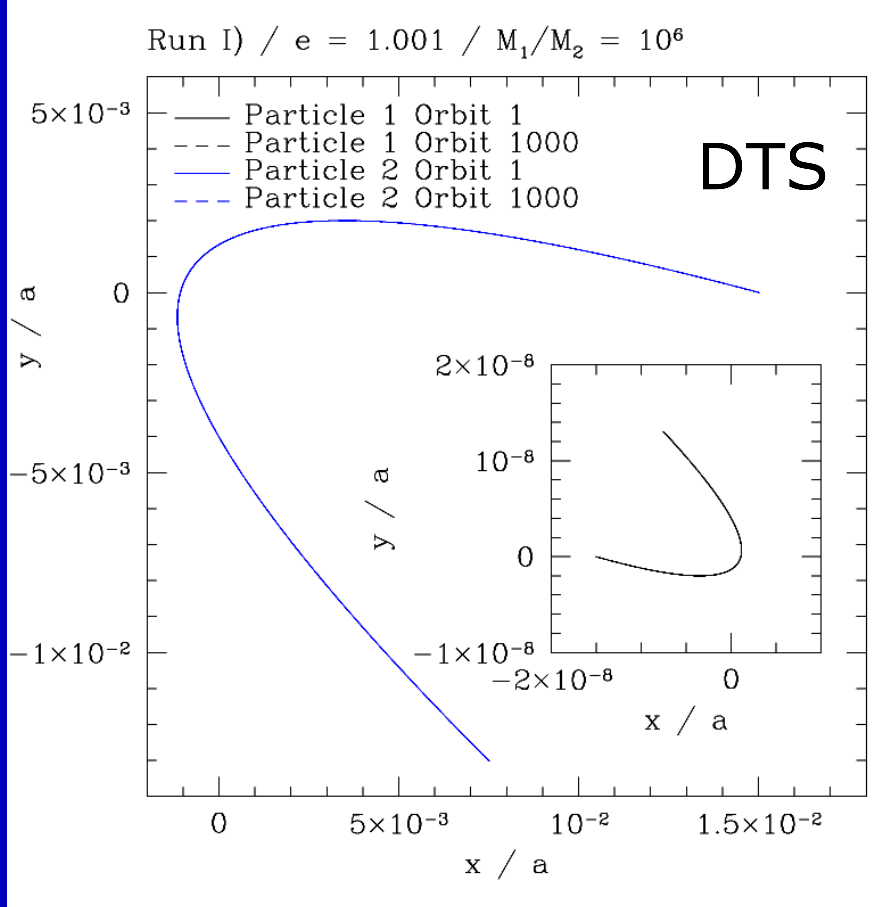
Cosmological Run





2-body Orbits I

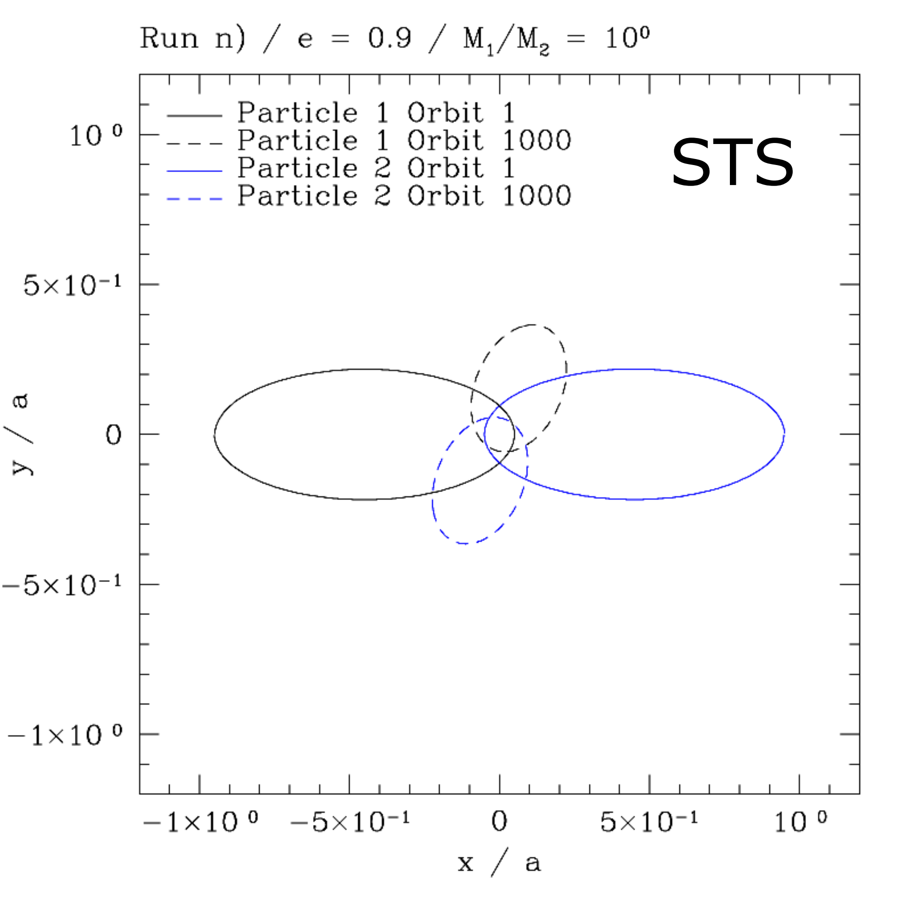


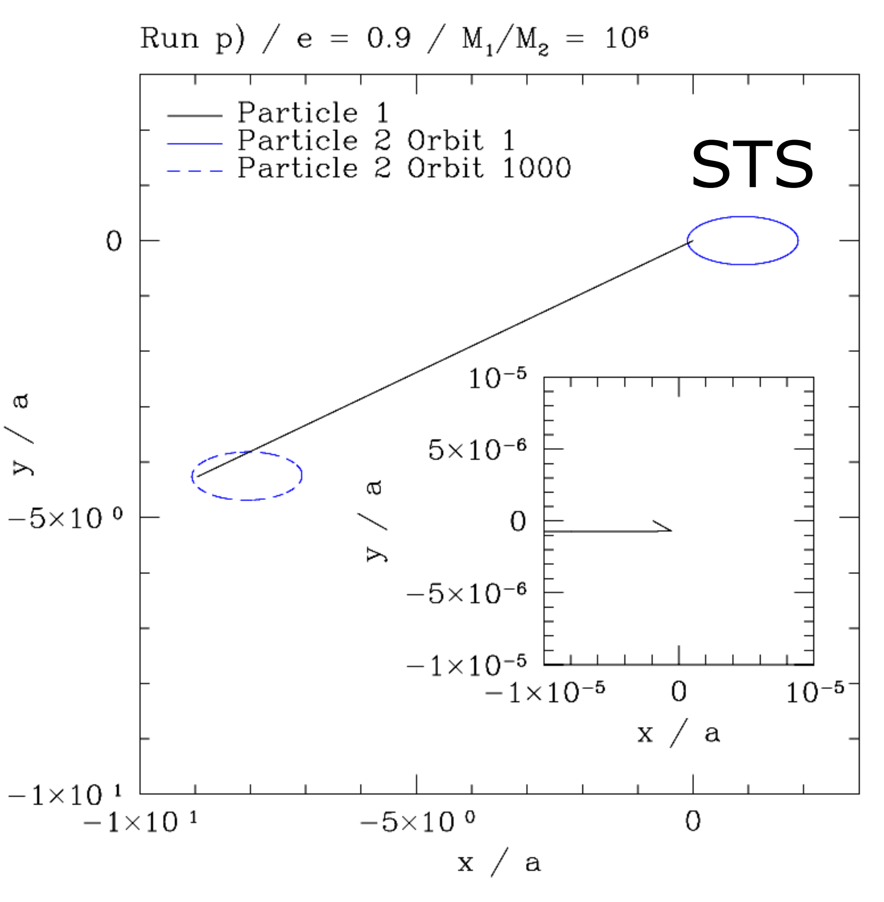


$\Delta E/|E_0| \approx +2.8\times10^{-3}$

$\Delta E/|E_0| \approx -1.3\times10^{-2}$

2-body Orbits II



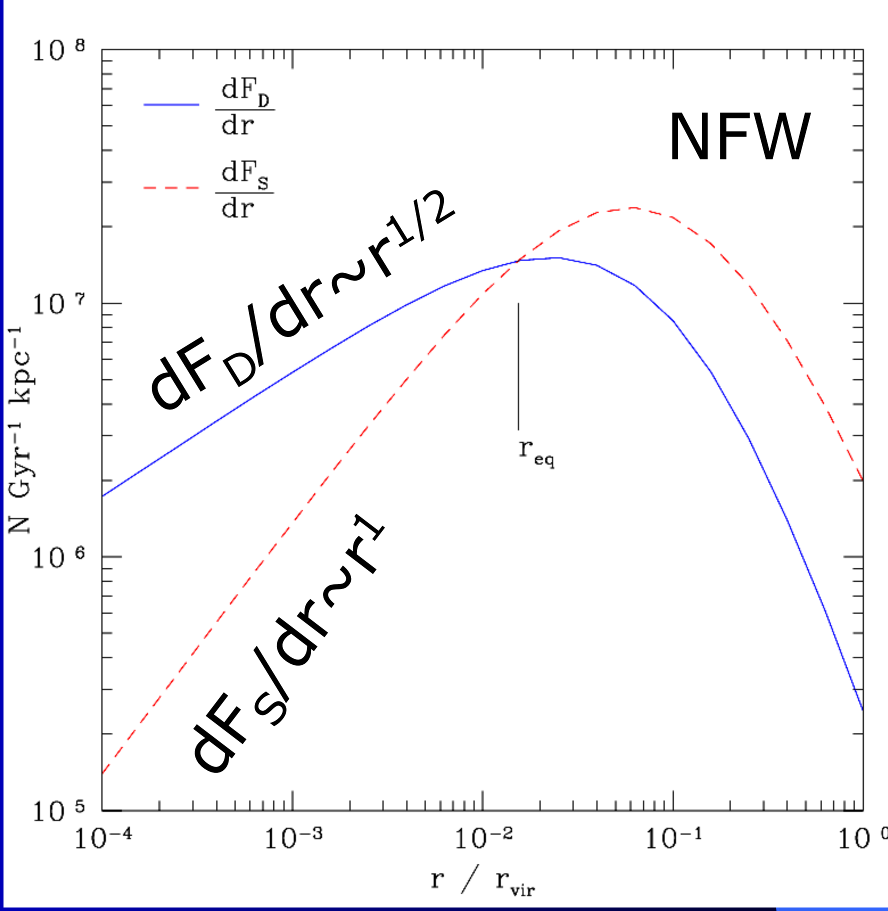


$\Delta E/|E_0|\approx$-1.3

$\Delta E/|E_0|\approx$+1.9

Efficiency

- Force evaluations per Gyr
- $F_D = \int dF_D dr \sim N$
- $F_S = \int dF_S dr \sim N^q$
- q depends on scaling of ε
- $\varepsilon \sim N^{-1/3}$
  $\Rightarrow q = 7/6$



Conclusions I

- DTS does not directly depend on artificial parameters like softening
- It gives physically correct time-steps in haloes with arbitrary central slope

Conclusions II

- It is faster in high resolution simulations
- Orbits with $e\rightarrow 1$ are followed correctly
- It allows to follow complex dynamical systems where scattering events are important