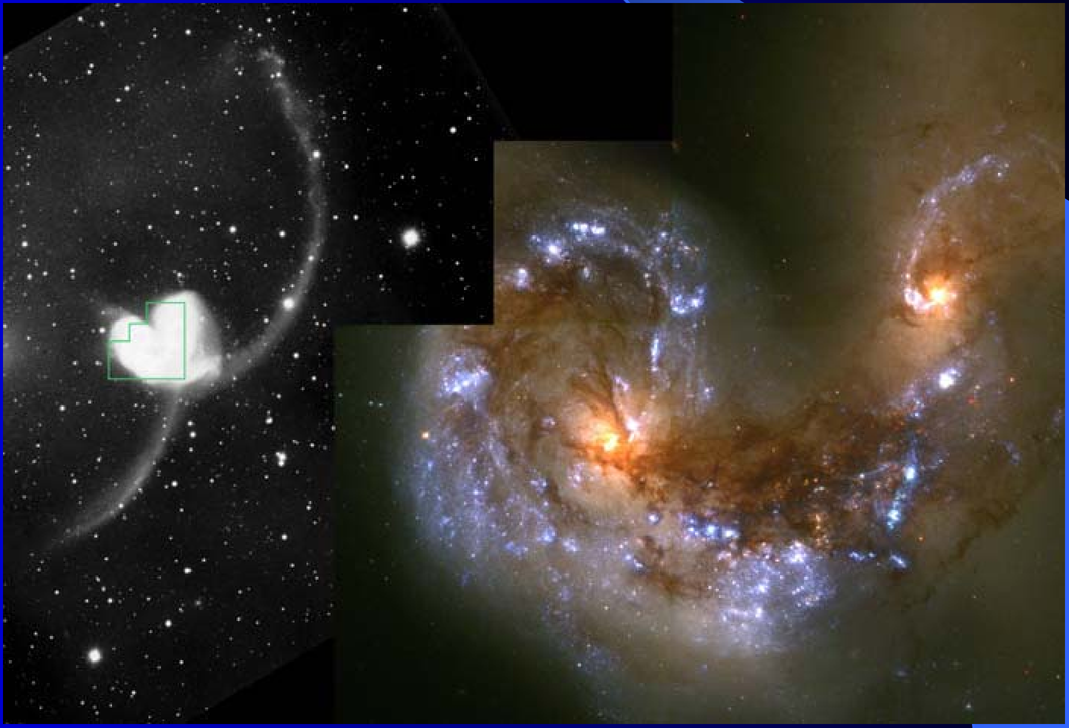
Collisionless and Collisional Dynamics in Astrophysics



Marcel Zemp
Thesis Talk
7. September 2006

Content

- Initial motivation
- Multi-mass halo models
- An optimum time-stepping scheme
- Applications
- Conclusions, Perspective and Plans

Initial motivation

- Study mergers of galaxies with super-massive black holes
- Involves collisionless and collisional dynamics
- Spatial range: 1 pc - 1 Mpc
- Temporal range: 1 yr – 1 Gyr

Multi-mass halo models

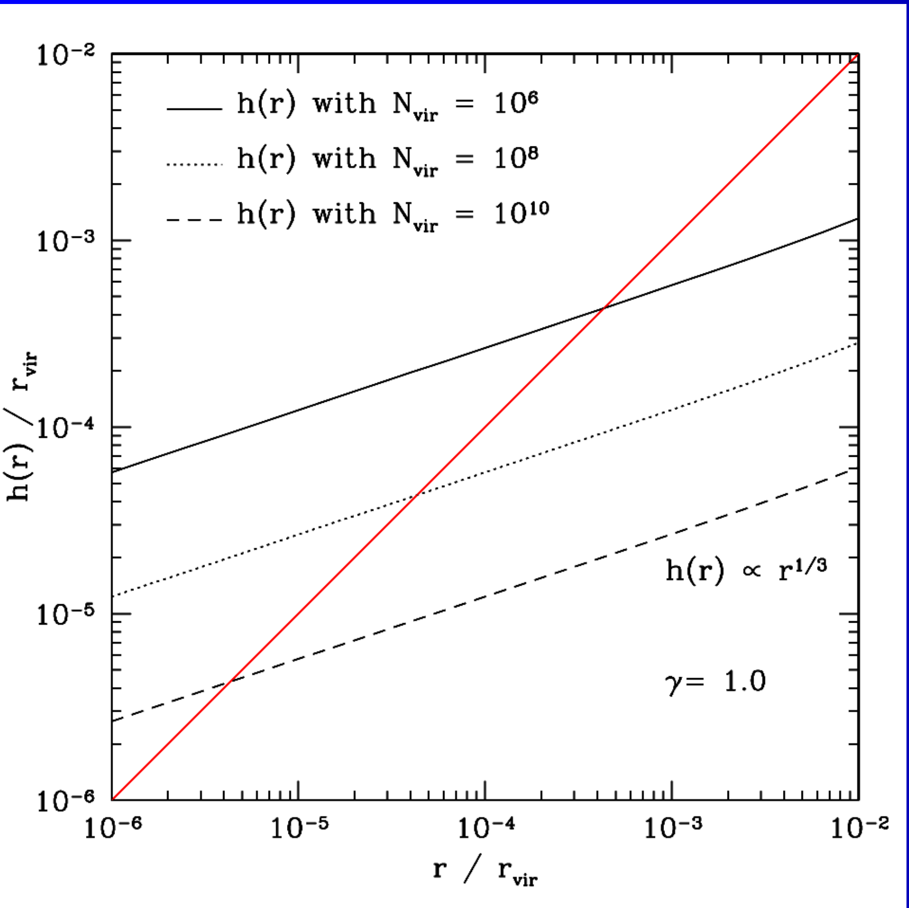
Multi-mass halo models

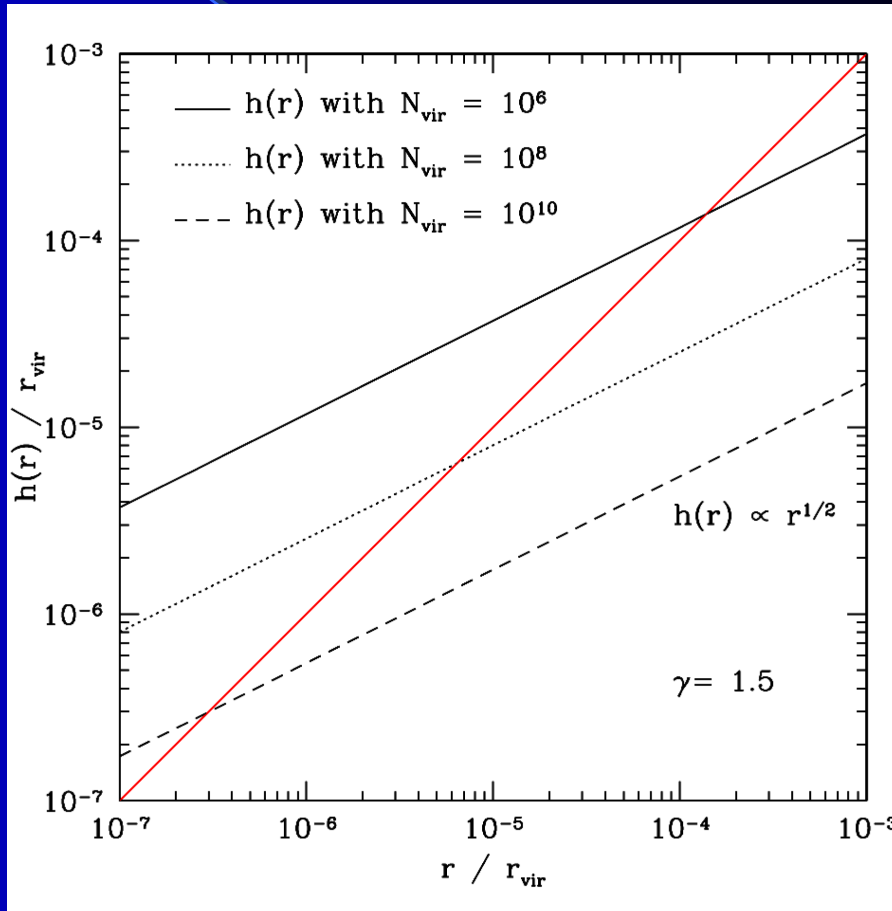
- General density profiles

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^{\gamma}\left(1+\left(\frac{r}{r_s}\right)^{\alpha}\right)^{\left(\frac{\beta-\gamma}{\alpha}\right)}}$$

- Dark matter haloes are well fitted by $(\alpha,\beta,\gamma) \approx (1,3,1)$

Mean particle separation





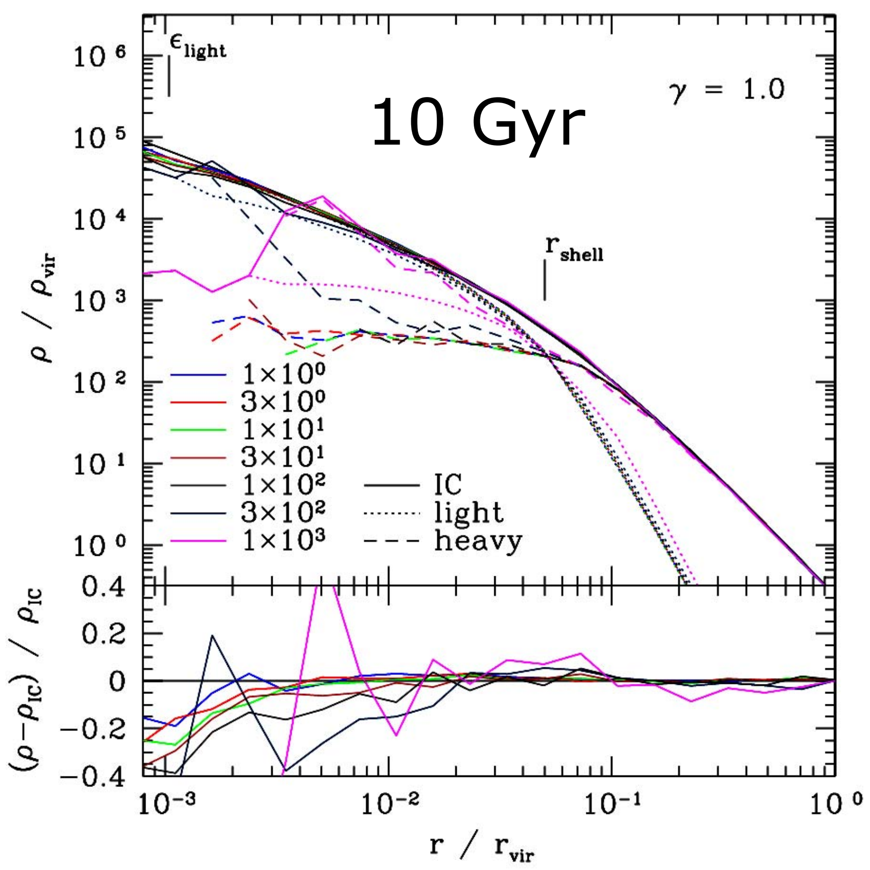
$$h(r) \equiv \sqrt[3]{m / \rho(r)}$$

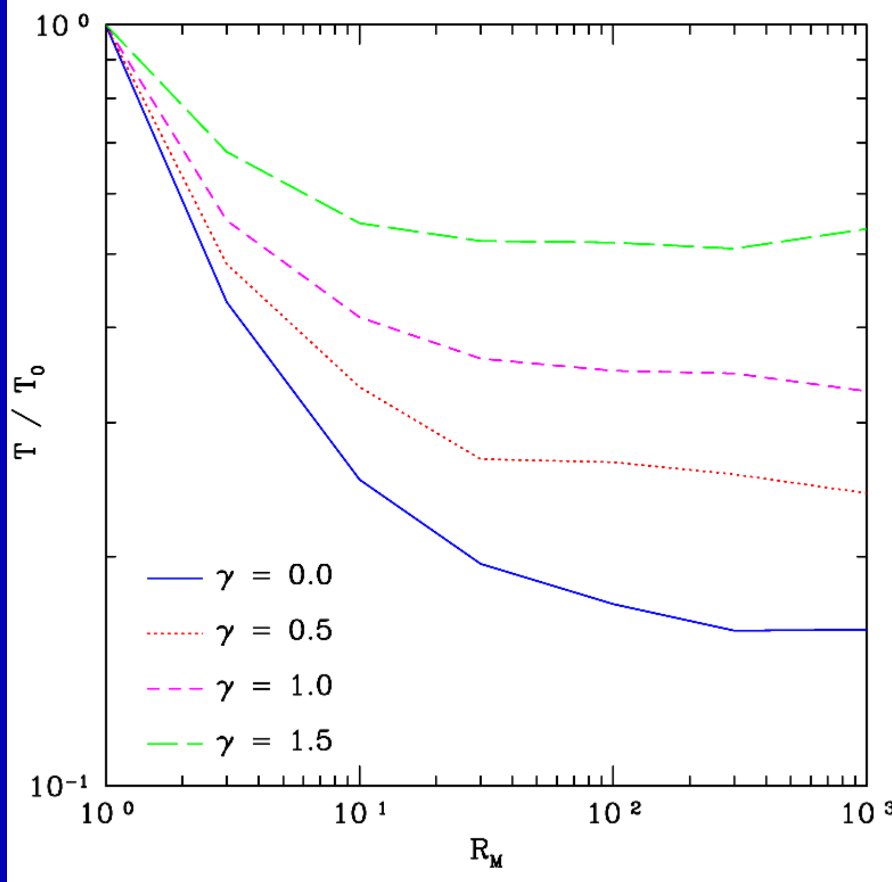
Multi-mass halo models

- Halo is represented by shells with different particle masses
  $\Rightarrow$ light particles in the centre
  $\Rightarrow$ heavy particles in the outskirts

- Numerical Problems
  $\Rightarrow$ heating of light particles
  $\Rightarrow$ mass segregation

Mass ratio R_M





An optimum time-stepping scheme

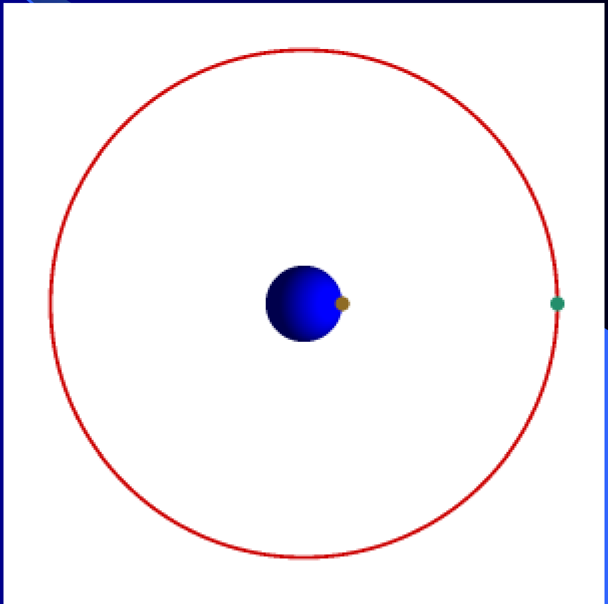
General Idea

- What is the natural time-scale?

Dynamical Time

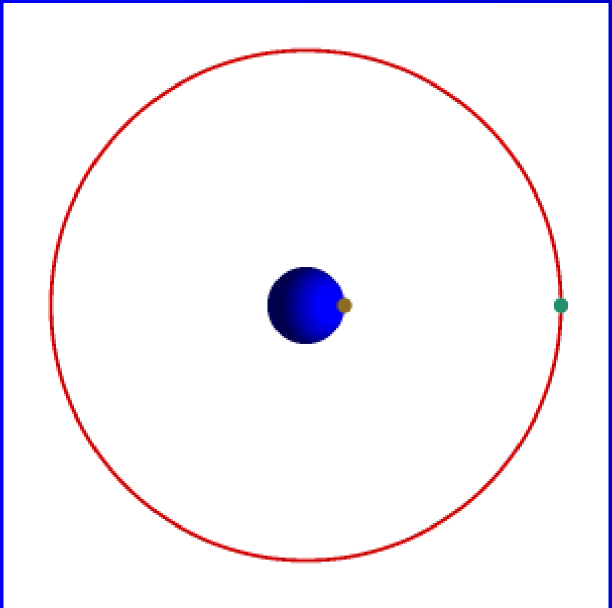
$$T_{dyn}(r) = \frac{2\pi}{\sqrt{G\rho_{enc}(r)}}$$

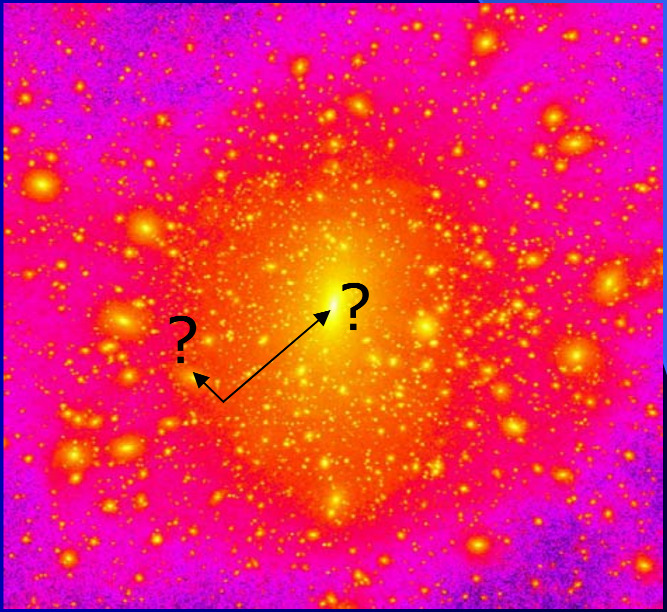
$$\rho_{enc}(r) \equiv M(r) / r^3$$



Dynamical Time-Stepping

- Natural choice $\Delta T \sim T_{dyn}$
- But what sets the dynamical time?





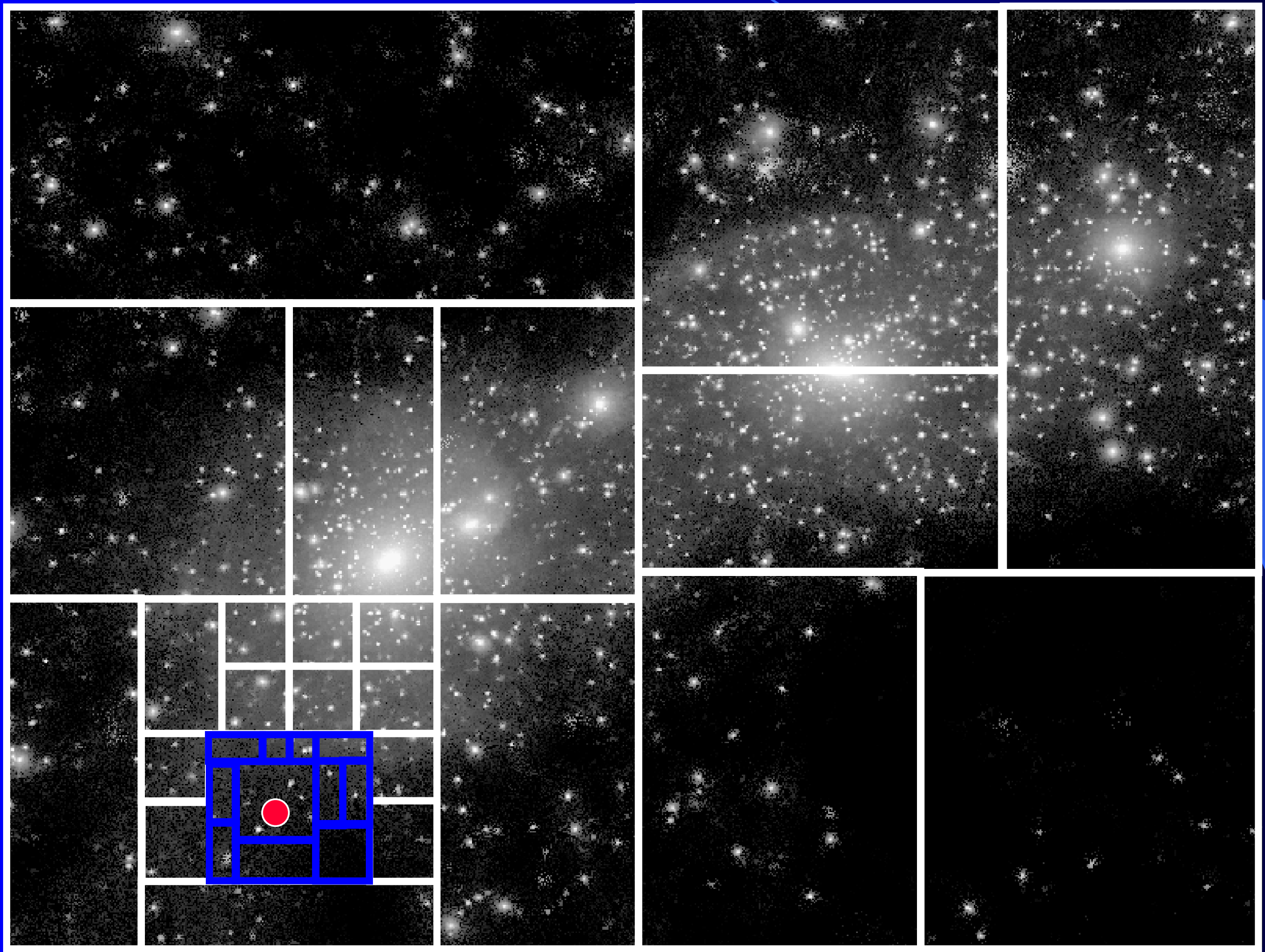
Two Regimes

- Mean field regime
  - global potential
  - no short range contributions

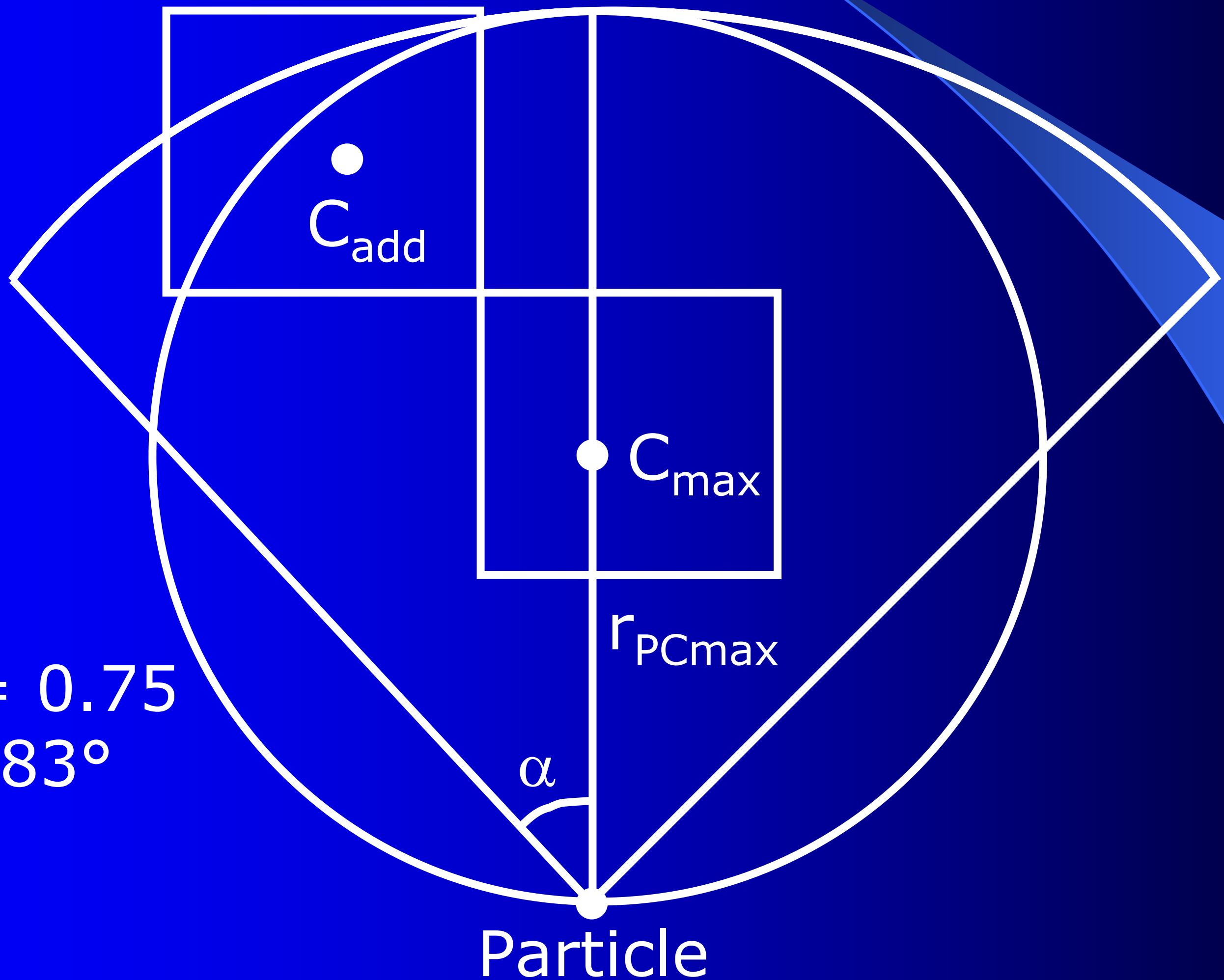
  $\Rightarrow \rho_{enc}$ set by global structures
- Gravitational scattering regime
  - large angel scattering
  - 2-body orbits with $e \to 1$

  $\Rightarrow \rho_{enc}$ set by local structures

Mean Field



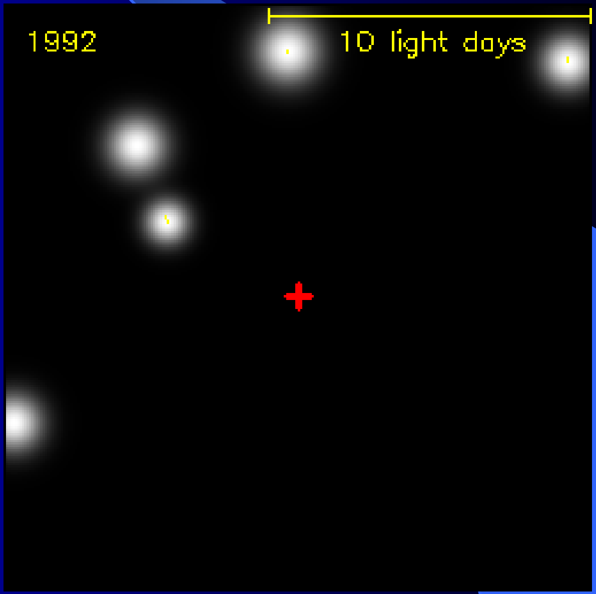
Mean Field Algorithm



$\cos(\alpha) = 0.75$
$\Rightarrow 2\alpha \approx 83°$

Gravitational Scattering

- We also want to follow 2-body orbits with $e \to 1$ correctly (elliptic & hyperbolic)
- gravitational scattering
- new time-stepping is starting point but…



Eccentricity Correction

- PKDGRAV: Kick-Drift-Kick Leapfrog
- Appr

$$E_2^{peri} = \Delta T^2 H_2 = \frac{1}{24}\frac{(1+2e)}{(1-e)}\frac{\eta_D^2 G M_1 M_2}{a}$$

  p
  periapsis
  symmetrised time-step

GS Algorithm

- Use symmetrised ρ_{enc}!

$$\rho_{enc,GS} = \frac{(1+2e)}{(1-e)} \frac{M_P + M_I}{r_{PP}^3}$$

- Similar to expression for orbital time in 2-body problem

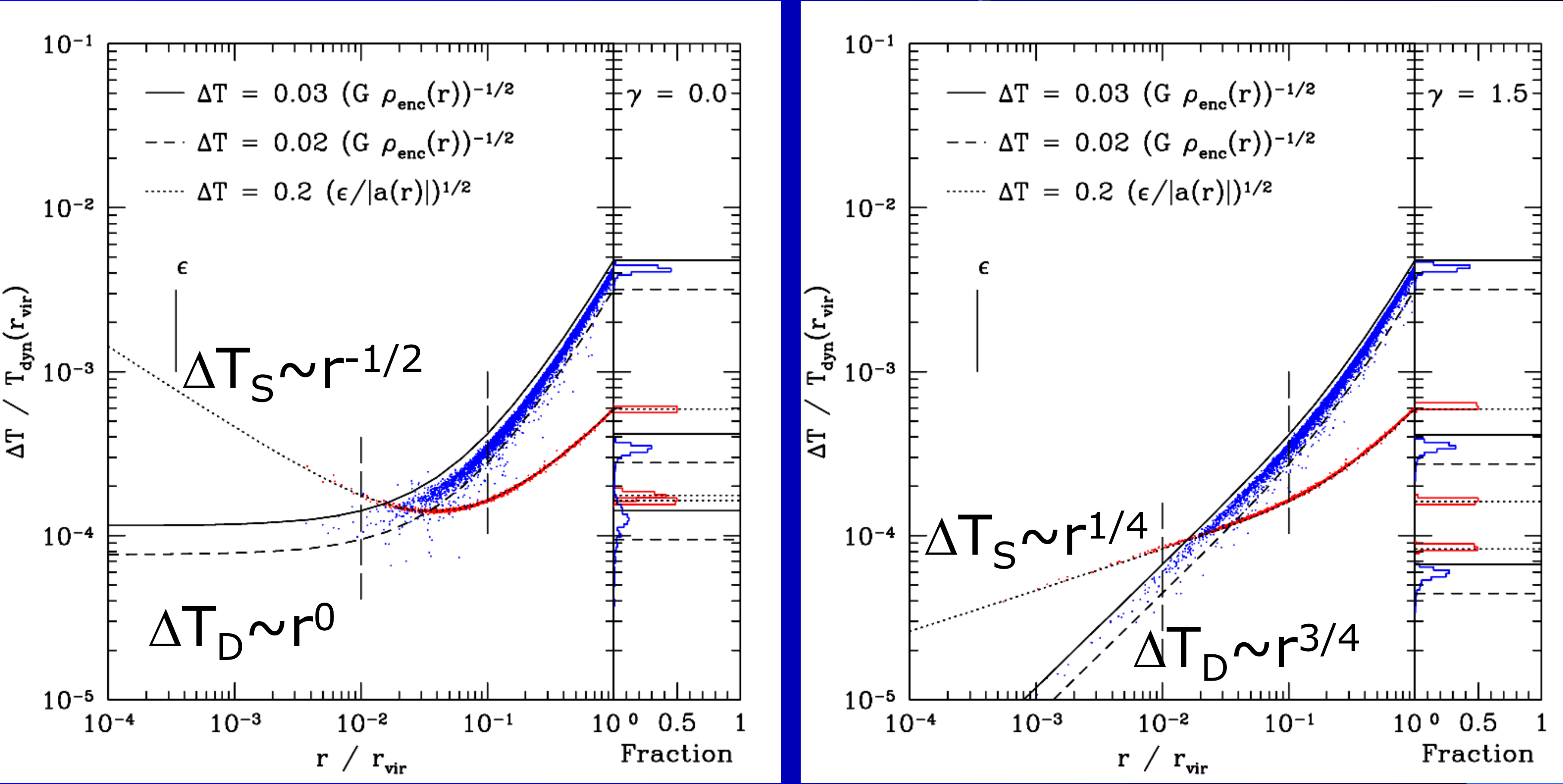
Standard Criterion

- Standard criterion used in PKDGRAV and GADGET

$$\Delta T = \frac{T_0}{2^n} \leq \eta_s \sqrt{\frac{\varepsilon}{a(r)}}$$

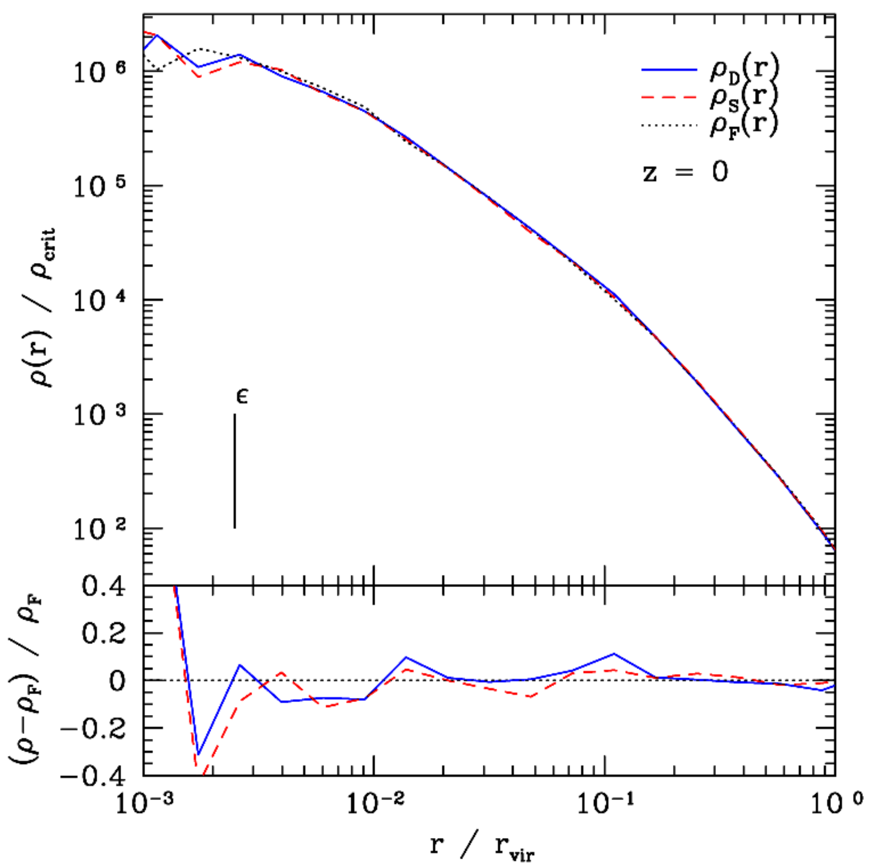
- Ad-hoc criterion
  $\Rightarrow$ no physical motivation!
- Dependence on artificial simulation parameter softening length ε

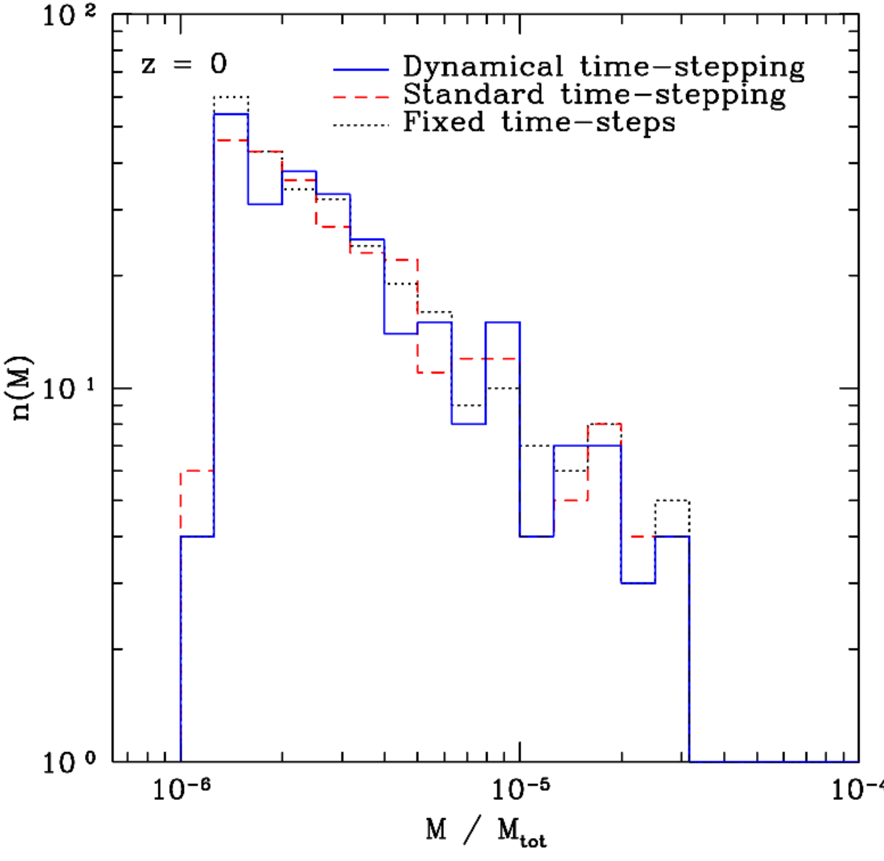
Comparison



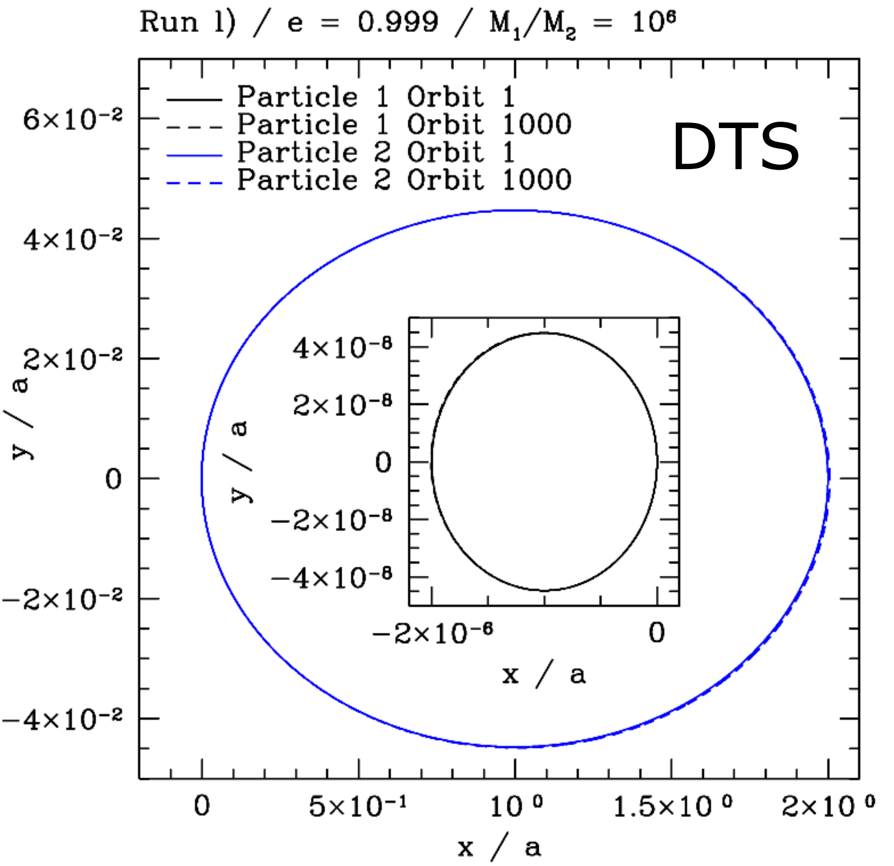
ΔT = 0.03 (G ρenc(r))^-1/2
ΔT = 0.02 (G ρenc(r))^-1/2
ΔT = 0.2 (ε/|a(r)|)^1/2
γ = 0.0
γ = 1.5
ε
ΔT / Tdyn(rvir)
r / rvir
Fraction
ΔTS~r^-1/2
ΔTD~r^0
ΔTS~r^1/4
ΔTD~r^3/4

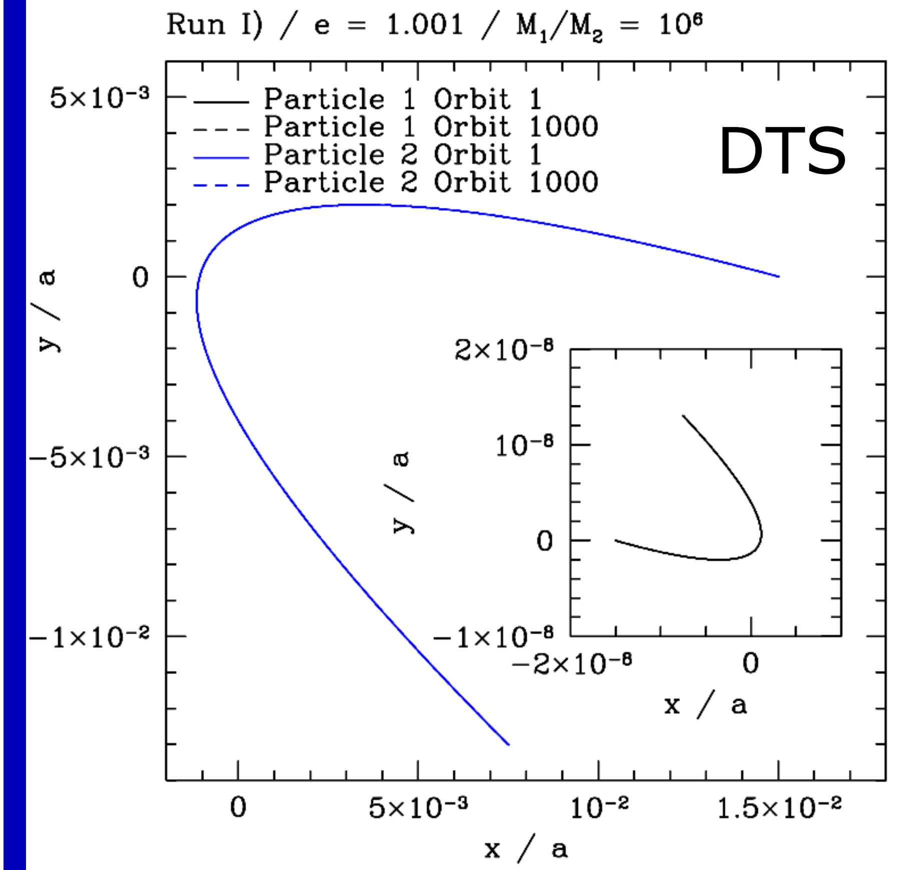
Cosmological Run





2-body Orbits I

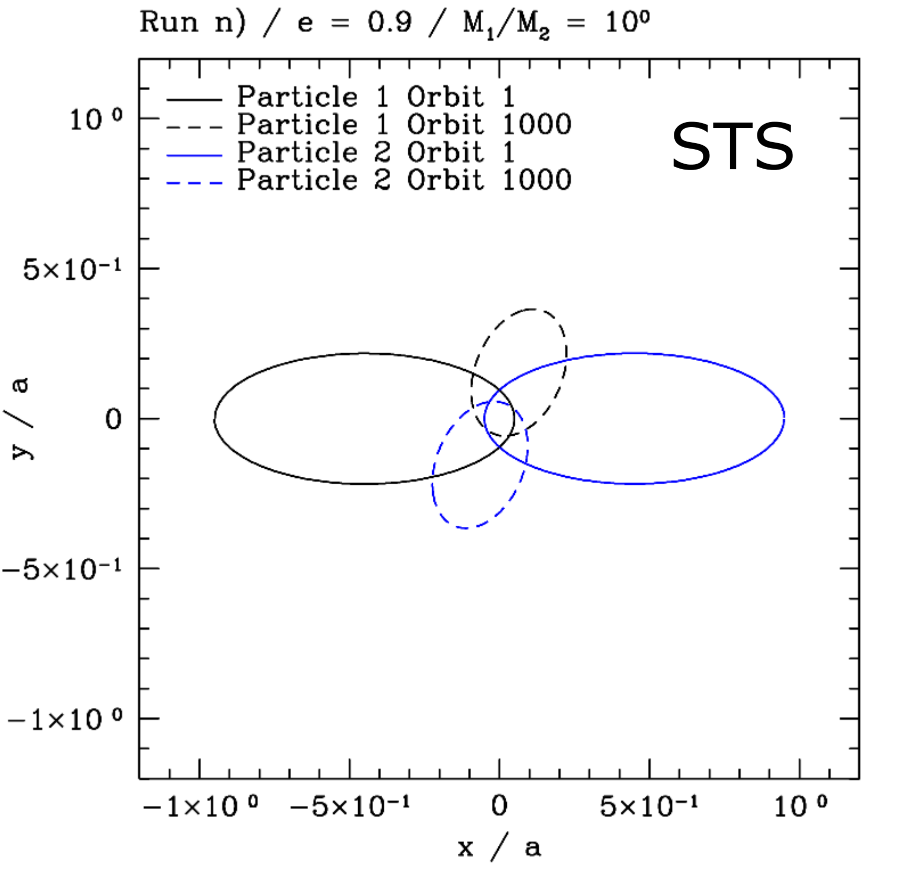


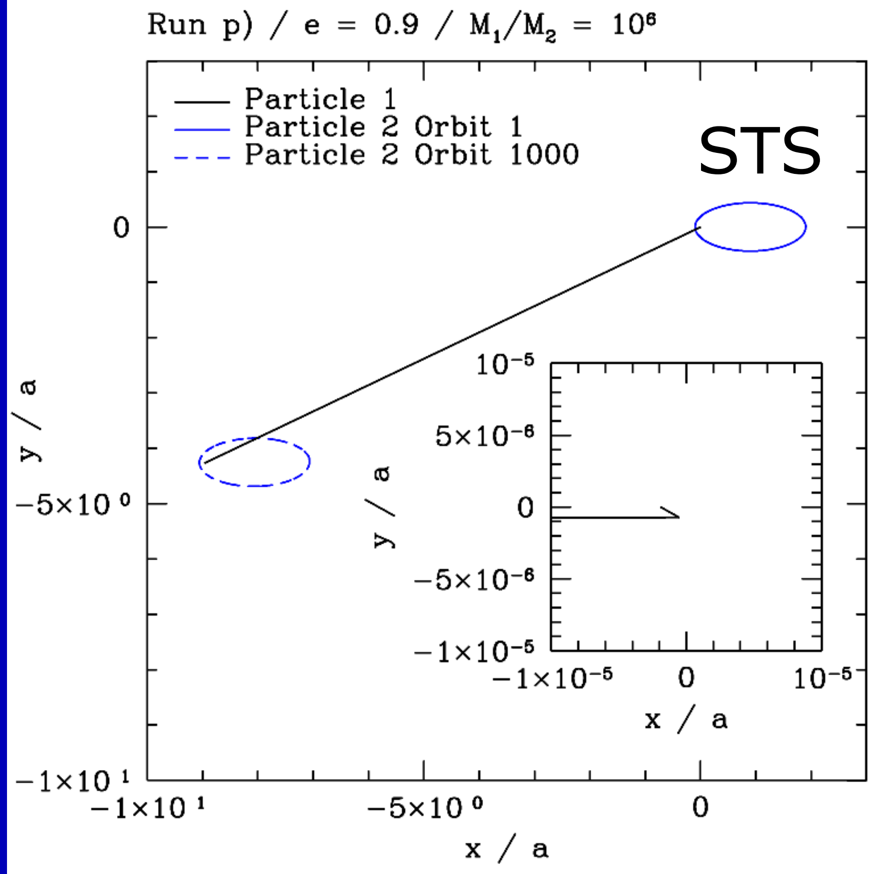


$\Delta E/|E_0| \approx +2.8\times10^{-3}$

$\Delta E/|E_0| \approx -1.3\times10^{-2}$

2-body Orbits II



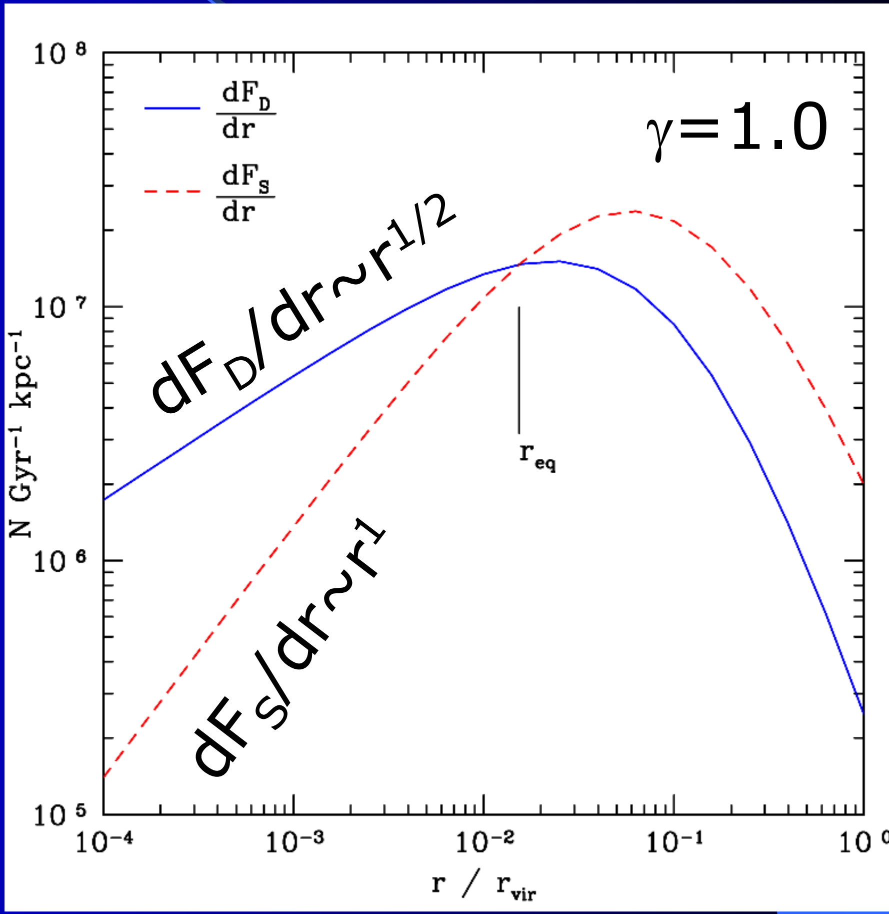


$\Delta E/|E_0| \approx -1.3$

$\Delta E/|E_0| \approx +1.9$

Efficiency

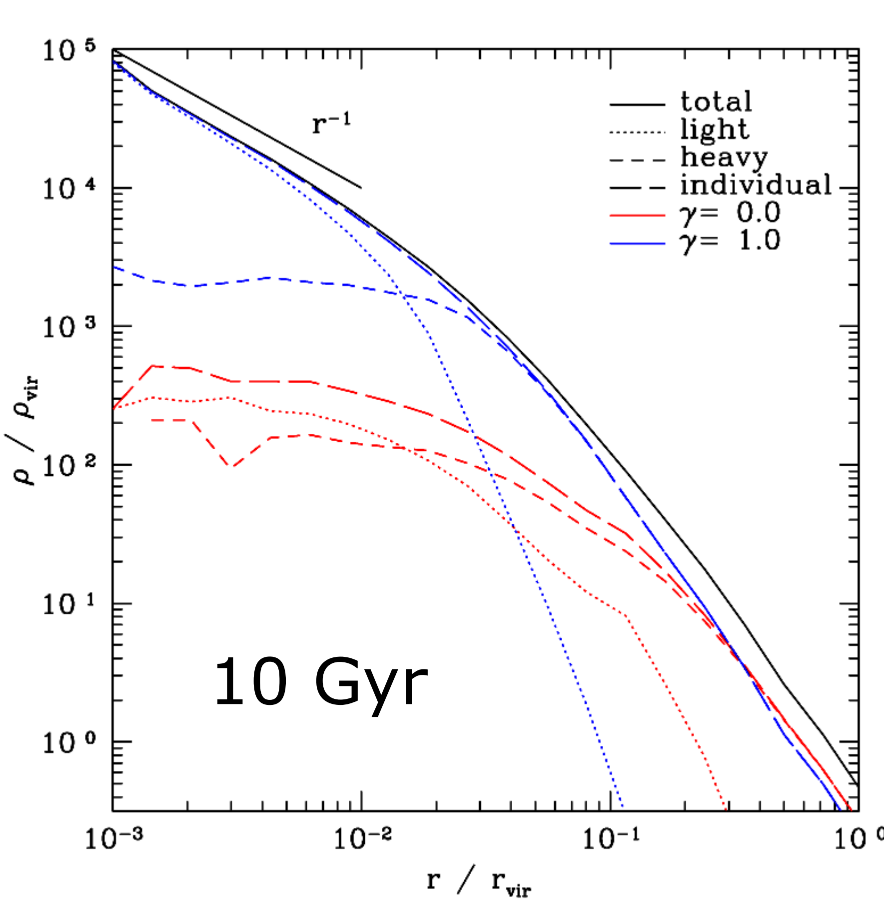
- Force evaluations per Gyr
- $F_D = \int dF_D dr \sim N$
- $F_S = \int dF_S dr \sim N^q$
- q depends on scaling of ε
- $\varepsilon \sim N^{-1/3}$ $\Rightarrow q = 7/6$



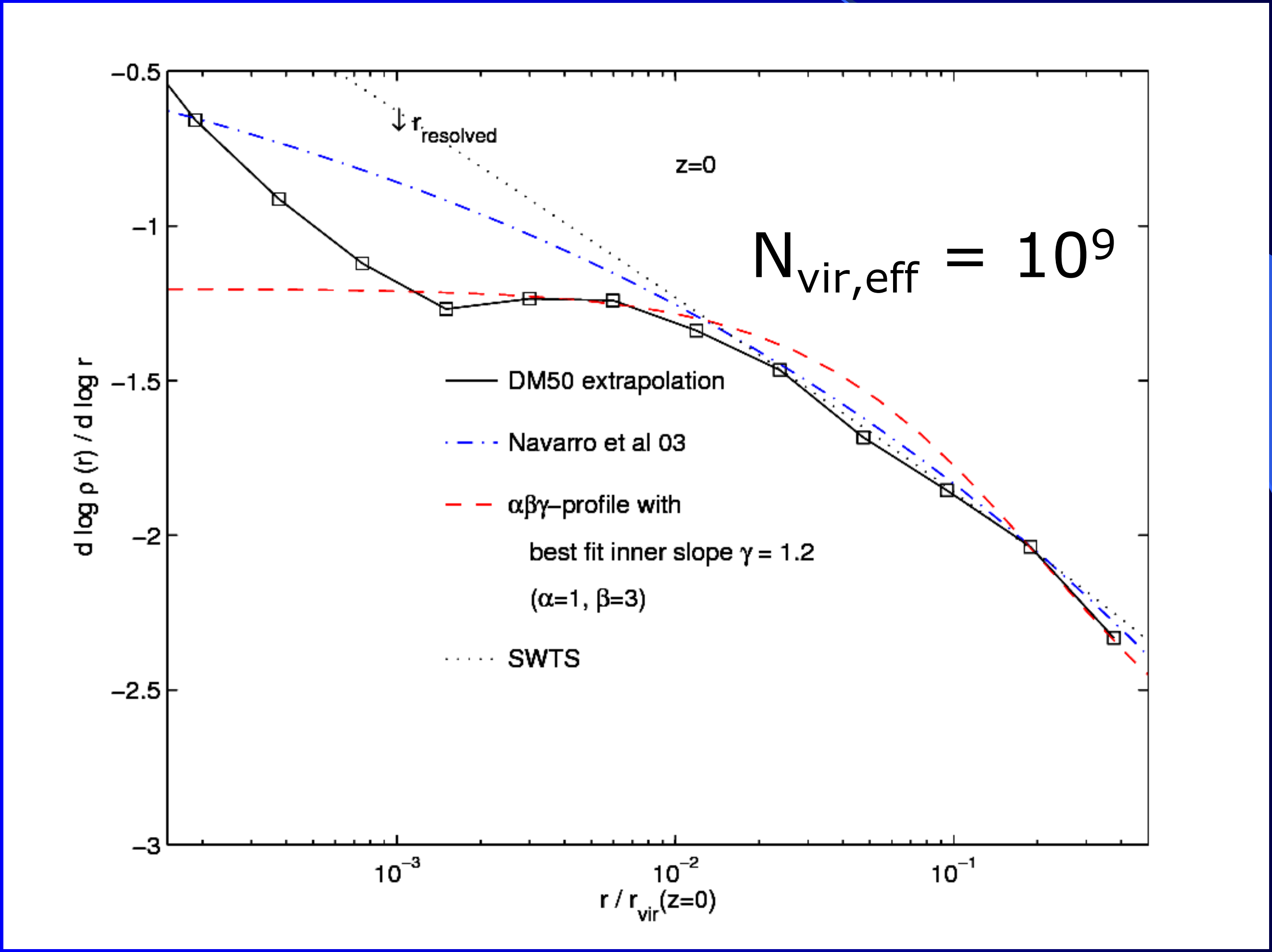
Applications

Cusp-core mergers

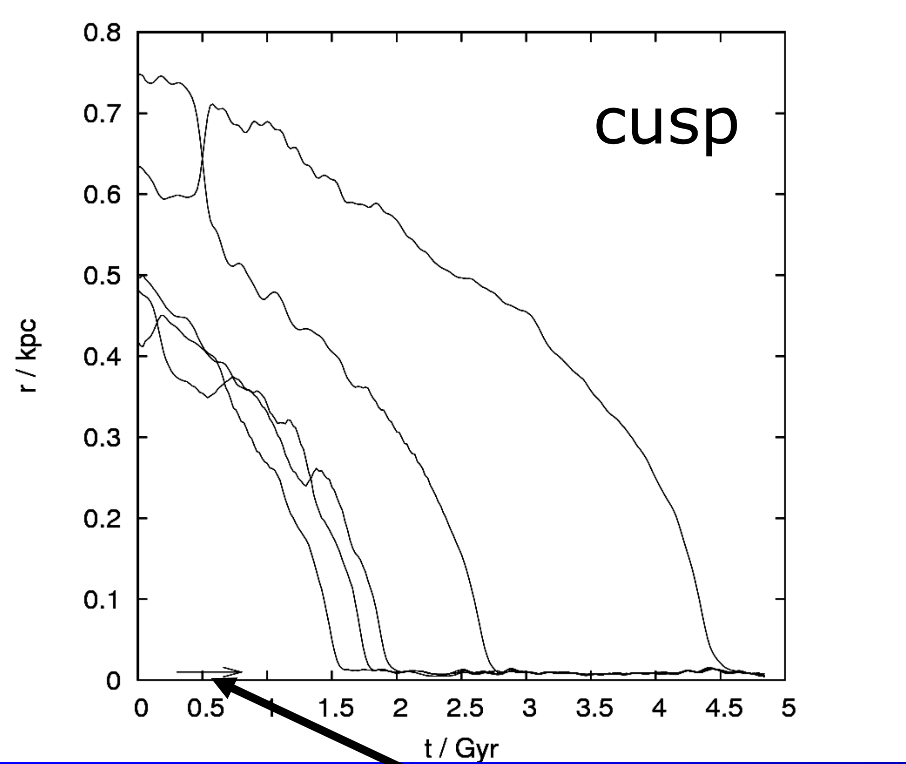
- Cusp-core merger: Steepest profile is preserved

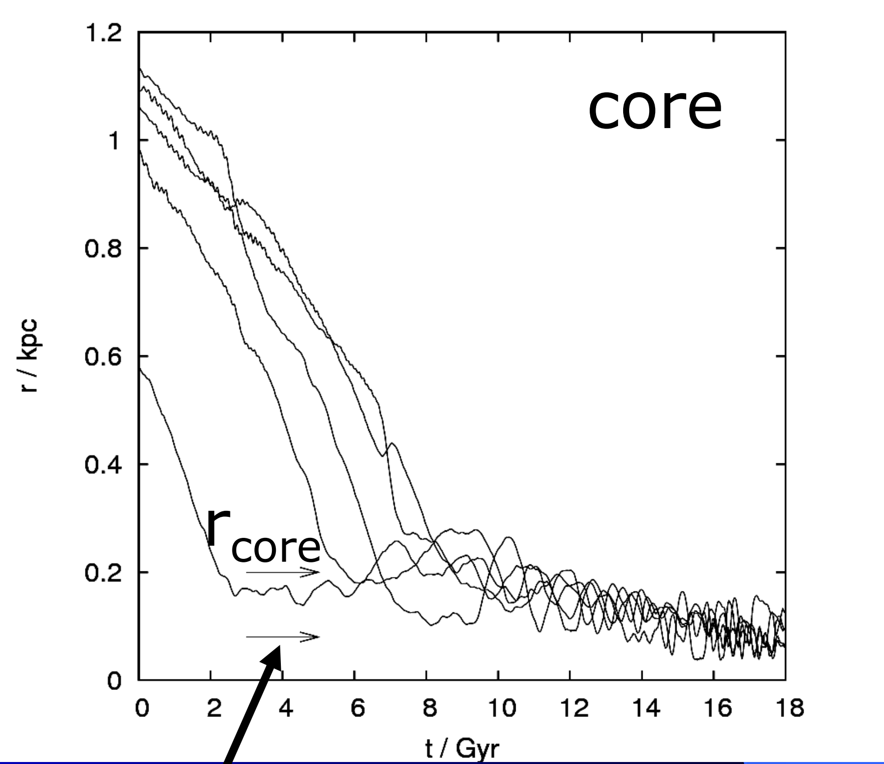


Cusps in CDM haloes



Sinking GCs





$M_{GC} = M(r)$

Conclusions, Perspective and Plans

Conclusions I

- Multi-mass technique leads to a significant gain in computer run time
- DTS gives physically correct time-steps in haloes with arbitrary central slope
- DTS does not directly depend on artificial parameters like softening

Conclusions II

- DTS is faster in high resolution simulations
- Orbits with $e \to 1$ are followed correctly
- DTS allows to follow complex dynamical systems where scattering events are important

Perspective and Plans

- Mergers of galaxies with super-massive central black holes
- Ejection of hyper-velocity stars from galaxy centres
- High-resolution cosmological structure formation simulations